Inclusion of Matter in Inhomogeneous Loop Quantum Cosmology

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Sep 1st, 2011 - ERE2011 - UCM
Introduction

- Loop Quantum Cosmology (LQC)
  - Quantum approach for cosmological systems inspired by the Loop Quantum Gravity (LQG)
  - Satisfactory quantization of several homogeneous cosmological models
  - New quantum phenomenology $\Rightarrow$ Resolution of initial singularity

- Hybrid quantization: Inhomogeneous models
  - Reduced model with only global constraints
  - It assumes that the most relevant effects of loop quantum geometry are in the homogeneous degrees of freedom
  - Combine LQC quantization for this homogeneous sector with a Fock quantization for inhomogeneities

- First model studied: vacuum Gowdy model with the three torus topology [Garay, Martín-Benito, Mena Marugán, '08]
Inclusion of a massless scalar field in the Gowdy $\mathbb{T}^3$ model
- Minimally coupled
- Same symmetries of the geometry

Motivation
- Inclusion of matter inhomogeneities in LQC.
- Study of more realistic models, closer to the observed universe.
- Scenario in which one can study some interesting features
  - Quantum effects of the inhomogeneities and the anisotropies on an FRW background.
  - Robustness of the Big Bounce scenario of LQC.
  - Changes in the evolution of the matter inhomogeneities due to quantum geometry effects.
  - *projection* to more symmetric quantum models.
Classical Settings

- Reduced phase space
  - Homogeneous sector: Bianchi I + homogeneous massless scalar field $\phi$
  - Inhomogeneous sector: Matter inhomogeneities and gravitational waves (propagating in $\theta \in S^1$)

- Ashtekar-Barbero variables for Bianchi I
  - $su(2)$ connection: $c^j$; densitized triad: $p_j \quad i \in \{\theta, \sigma, \delta\}$

- Satisfactory Fock quantization of the inhomogeneities:
  - Unitary dynamics + Vacuum invariant under $S^1$ translations.
  - Parametrization of the matter $\varphi$ and gravitational $\xi$ inhomogeneities
  - Creation-annihilation variables (free m. s. f.): $\left( a_m^{(\alpha)*}, a_m^{(\alpha)} \right), \alpha = \xi, \varphi$

- Two global constraints remain:
  - Diffeomorphism constraint: $C_\theta = C_\theta^\xi + C_\theta^\varphi$
  - Densitized Hamiltonian constraint: $C = C_{\text{hom}} + C_{\text{inh}}$
Hybrid Quantization: Kinematics

**Kinematical Hilbert space**

\[ \mathcal{H}_{\text{kin}} = \mathcal{H}^{\text{hom}}_{\text{kin}} \otimes \mathcal{H}^{\text{inh}}_{\text{kin}} = \mathcal{H}^{\text{BI}}_{\text{kin}} \otimes L^2(\mathbb{R}, d\phi) \otimes \mathcal{F}^\xi \otimes \mathcal{F}^\varphi \]

- **Fock Spaces** \( \mathcal{F}^\alpha \):
  - Creation-annihilation operators:
    - \( a_m^{(\alpha)*}, a_m^{(\alpha)} \rightarrow \hat{a}_m^{(\alpha)*}, \hat{a}_m^{(\alpha)} \)
  - n-particle states:
    \[ |n^{\alpha}\rangle = |\ldots, n^{\alpha}_{-m}, \ldots, n^{\alpha}_m, \ldots\rangle, n^{\alpha}_m \in \mathbb{N}, \sum_m n^{\alpha}_m < \infty \]

- **\( \mathcal{H}^{\text{hom-mat}}_{\text{kin}} = L^2(\mathbb{R}, d\phi) \):**
  - Standard Schrödinger quantization:
    \[ \hat{\phi}, \hat{p}_\phi = -i\hbar\partial_\phi \]

- **Bianchi I kinematical Hilbert space:**
  - LQC: Triads \( p_i \) and holonomies of connections \( N_{\mu_i}(c_i) \).
  - Improved dynamics: minimum length, \( \bar{\mu}_j \), in the holonomies.
  - \( \mathcal{H}^{\text{BI}}_{\text{kin}} = \text{span}\{|\lambda_\theta, \lambda_\sigma, v\rangle : \lambda_\theta, \lambda_\sigma, v \in \mathbb{R}\} \)
  - Discrete inner product:
    \[ \langle \lambda_\theta, \lambda_\sigma, v | \lambda_\theta', \lambda_\sigma', v' \rangle = \delta_{\lambda_\theta, \lambda_\theta'} \delta_{\lambda_\sigma, \lambda_\sigma'} \delta_{vv'} \]
  - \( \hat{N}_{\pm\mu_i} \): Scale \( \lambda_i \) such that shift \( v \) in \( \pm 1 \);
    \[ \hat{p}_j : p_j \propto \text{sgn}(\lambda_j)\lambda_j^2 \]
Operators on the inhomogeneous Hilbert space

- Diffeomorphism constraint operator

\[
\hat{C}_\theta = \sum_{m=1}^{\infty} m \left( \hat{X}_m^\xi + \hat{X}_m^\varphi \right), \quad \hat{X}_m^\alpha = \hat{a}_m^\dagger \hat{a}_m^\alpha - \hat{a}_{-m}^\dagger \hat{a}_{-m}^\alpha.
\]

\[
\hat{C}_\theta |n^\xi\rangle \otimes |n^\varphi\rangle \Rightarrow \sum_{m=1}^{\infty} m (X_m^\xi + X_m^\varphi) = 0, \quad X_m^\alpha = n_m^\alpha - n_{-m}^\alpha.
\]

- \( \mathcal{F}_p \equiv \) proper subspace of \( \mathcal{F}_\xi \otimes \mathcal{F}_\varphi. \)

- Operators in \( \hat{C}_{\text{inh}} \)

\[
\hat{H}_0 = \sum_{\alpha \in \{\xi, \varphi\}} \sum_{m=1}^{\infty} m \hat{N}_m^\alpha, \quad \hat{N}_m^\alpha = \hat{a}_m^\dagger \hat{a}_m^\alpha + \hat{a}_{-m}^\dagger \hat{a}_{-m}^\alpha.
\]

\[
\hat{H}_{\text{int}} = \sum_{\alpha \in \{\xi, \varphi\}} \sum_{m=1}^{\infty} \frac{1}{m} \left( \hat{N}_m^\alpha + \hat{a}_m^\dagger \hat{a}_{-m}^\alpha + \hat{a}_m^\alpha \hat{a}_{-m}^\dagger \right).
\]
Hamiltonian constraint operator $\hat{\mathcal{C}} = \hat{\mathcal{C}}_{\text{hom}} + \hat{\mathcal{C}}_{\text{inh}}$

- $\hat{\mathcal{C}}_{\text{hom}} = -\sum_{i\neq j} \sum_{j} \frac{\hat{\Theta}_i \hat{\Theta}_j}{16\pi G \gamma^2} - \frac{\hbar^2}{2} \left[ \frac{\partial}{\partial \phi} \right]^2$, $i, j \in \{\theta, \delta, \sigma\}$.

- $\hat{\mathcal{C}}_{\text{inh}} = 2\pi \hbar |p_{\theta}| \hat{H}_0 + \hbar \left[ \frac{1}{|p_{\theta}|^\frac{1}{4}} \right]^2 \left( \hat{\Theta}_\delta + \hat{\Theta}_\sigma \right)^2 \left[ \frac{1}{|p_{\theta}|^\frac{1}{4}} \right]^2 \hat{H}_{\text{int}}$.

- $\hat{\Theta}_i = \hat{c}_i \hat{p}_i = i \pi G \hbar \sqrt{|v|} \left[ (\hat{N}_{-2\mu_i} - \hat{N}_{2\mu_i}) \text{sgn}(p_i) + \text{sgn}(p_i) (\hat{N}_{-2\mu_i} - \hat{N}_{2\mu_i}) \right] \sqrt{|v|}$

Symmetric factor ordering:

- Triad operators: $v = 0$ states decouple (kin. singularity resolution)
- $\hat{\Theta}_j$ operators do not mix states with different sign of $\lambda_\theta, \lambda_\sigma, v$.

- $\tilde{\mathcal{H}}_{\text{kin}}^{\text{BI}}$: states such that $\lambda_\theta, \lambda_\sigma, v > 0 \Rightarrow \Lambda_\theta = \log \lambda_\theta, \Lambda_\sigma = \log \lambda_\sigma$.

Superselection sectors:

- in $v$: $v \in \mathcal{L}_\epsilon = \{\epsilon + 4k; \ k \in \mathbb{N}\}, \ \epsilon \in (0, 4]$  
- in $\Lambda_{\alpha}$: Given an initial $\Lambda_{\alpha}^* \Rightarrow \Lambda_{\alpha} = \Lambda_{\alpha}^* + z_\epsilon, \ z_\epsilon \in \mathbb{Z}_\epsilon$
Physical Hilbert space

- Action of the Hamiltonian constraint
  - The coefficients do not depend on $\Lambda_\sigma$
  - It is a difference equation in $v \Rightarrow$ evolution equation in $v$
  - The solutions can be determined by a set of initial data on the section of minimum homogeneous volume

- Physical Hilbert space $\mathcal{H}_{\text{phy}} \Leftrightarrow$ Hilbert space of initial data

\[ \mathcal{H}_p = \mathcal{H}_{\text{phys}}^{\text{BI}} \otimes L^2(\mathbb{R}, d\phi) \otimes \mathcal{F}_p \]

- $\mathcal{H}_{\text{phys}}^{\text{BI}} \equiv$ Physical Hilbert space of Bianchi I
The model is symmetric under the interchange of $\sigma$ and $\delta$ directions.

Classical solutions with local rotational symmetry (LRS)

General state: \[ |\Psi\rangle = \sum_{\Lambda, \Lambda, \lambda} |\Psi(\Lambda, \Lambda, \lambda)\rangle \otimes |\Lambda, \Lambda, \lambda\rangle \]

Projection map:

\[ |\Psi(\Lambda, \Lambda, \lambda)\rangle \rightarrow \sum_{\Lambda, \Lambda, \lambda} |\Psi(\Lambda, \Lambda, \lambda)\rangle \equiv |\psi(\Lambda, \lambda)\rangle \]

Quantum Gowdy Model \( \xrightarrow{projection} \) Quantum LRS-Gowdy Model

projection over $\Lambda$ to get the isotropic Gowdy model fails

There is no classical inhomogeneous and isotropic solutions.
Conclusions

- Satisfactory quantization of the Gowdy $T^3$ model with linearly polarized gravitational waves and a massless scalar field.
- Hybrid quantization applied as in the vacuum model.
- Inclusion of the matter field:
  - Classical isotropic solutions of the homogeneous sector.
  - Two “copies” of inhomogeneities (mathematically speaking).
  - Matter inhomogeneities in LQC.
- Same results as in the vacuum model.
  - Standard Fock quantization of the inhomogeneities is recovered.
  - Classical singularity resolved at the kinematical level.
- Study of the projection to more symmetric systems.
- Possibility of analyzing the effects of the anisotropies and the inhomogeneities on a flat FRW model. (Work in progress)
Thanks for your attention!