A Solar Abundance Study Using Recent Ti I Oscillator Strengths

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Summary. The f-values measured by Kühne et al. (1977) lead to a solar abundance of titanium, \( \log \varepsilon_{\text{Ti}} = 4.94 \pm 0.12 \) on the scale \( \log \varepsilon_{\text{H}} = 12 \). Comparison with abundances following from two independent sets of f-values (Ellis, 1976; Whaling et al., 1977) shows basic consistency of absolute scales.

The solar ratio Ti/Ca = \( 3.8 \times 10^{-3} (\pm 25\%) \) matches that found in chondritic meteorites of types C1 or C2, but contrasts with other types.

Further subjects of abundance analysis are the solar velocity field, constraints on departures from LTE, and the accuracy of the STFD oscillator strengths of Kurucz and Peytremann (1975).

Key words: oscillator strengths — solar Ti abundance — solar atmosphere

1. Introduction

The solar titanium abundances reported in literature (Table 1) exhibit a steady increase reminiscent of the iron abundance, though less drastic. As with iron, the principal reason is the progress made in identifying and eliminating errors of oscillator strengths. Has the sequence yet converged? The main purpose of the present study is to provide an additional abundance value based upon a new, independent set of experimental Ti I f-values (Kühne et al., 1977) (Sect. 2).

In an abundance study like this, each line, characterized by its equivalent width, provides an individual abundance value, given the solar model, f-value and other atomic parameters. Deviations of individual abundances may follow a specific pattern indicative of the particular input parameter that needs revision (see, e.g., Garz et al., 1969; Holweg, 1972). In Sections 3, 4, and 5 this is applied to Ti I. Use is made of two additional sets of f-values (Ellis, 1976; Kurucz and Peytremann, 1975).

2. Solar Abundance of Titanium

Computation of emergent solar line profiles was carried out using a code described by Baschek et al. (1966) and the solar model of Holweg and Müller (1974). Equivalent widths of reasonably unperturbed Fraunhofer lines were taken from Holweg (1967), additional lines were measured from the Preliminary Edition of the Kitt Peak Solar Atlas (Braunl and Testerman, 1972). The input data and abundances resulting from the f-values of Kühne et al. (1977) are summarized in Table 2 (see also Fig. 1a).

Line broadening by collisions with hydrogen (damping constant, \( \gamma_{\text{H}} \)) is of minor importance here. Deviations from pure van der Waals interaction were accounted for by an enhancement factor 2.5 applied to van der Waals damping, \( \gamma_{\text{v}} \), following from the dipole–dipole approximation (Unsöld, 1955, STFD mean square radii from Hofsass, 1975). Corrections of this order are suggested by solar studies (e.g., Holweg and Müller, 1974). Use of van der Waals broadening as a reference has been criticized (Roueff and Van Regemortel, 1971), but recent theoretical work (Brueckner, 1971; O’Mara, 1976) confirms that this scheme as well as the magnitude of the correction is realistic. The effect of the correction on the abundance is about 0.06 dex for line strengths of 60 mÅ. To eliminate residual uncertainties due to imperfect

Table 1. Solar titanium abundances, \( \log \varepsilon_{\text{Ti}} \), reported in the literature

<table>
<thead>
<tr>
<th>Reference</th>
<th>( \log \varepsilon_{\text{Ti}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldberg et al. (1960)</td>
<td>4.68</td>
</tr>
<tr>
<td>Müller and Mutschler (1964)</td>
<td>4.78</td>
</tr>
<tr>
<td>Warner (1968)</td>
<td>4.50</td>
</tr>
<tr>
<td>Grevesse (1970)</td>
<td>4.74</td>
</tr>
<tr>
<td>Wolnik and Berthel (1973)</td>
<td>4.83</td>
</tr>
<tr>
<td>Biémont (1974)</td>
<td>4.88</td>
</tr>
<tr>
<td>Bell et al. (1975)</td>
<td>4.74</td>
</tr>
<tr>
<td>Foy (1975)</td>
<td>4.82</td>
</tr>
<tr>
<td>Ellis (1976)</td>
<td>4.82</td>
</tr>
<tr>
<td>Whaling et al. (1977)</td>
<td>4.98</td>
</tr>
</tbody>
</table>

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knowledge of this factor we omit lines stronger than 50 mA from the determination of the final abundance.

The solar Ti abundance following from the remaining 13 Ti i lines of Table 2 is

\[ \log e_{\text{Ti}} = 4.94 \pm 0.10 \]

on the scale where \[ \log e_{\text{Ti}} = 12. \] The standard deviation of \[ \pm 0.10 \] reflects the uncertainty of relative \( f \)-values and equivalent widths. The overall uncertainty of the abundance relative to hydrogen, including that of the absolute scale and the solar model, is estimated to about \[ \pm 0.12. \] If the model used here is replaced by the HSRA (Gingerich et al., 1971) the abundance decreases by 0.08.

An illustrative test of present-day oscillator strengths will be to compare our result, \[ \log e_{\text{Ti}} = 4.94 \] with that following from two independent recent sets of experimental \( f \)-values and the same solar model. The measurements by Whaling et al. (1977) have 9 lines in common with our 13, yielding \[ \log e_{\text{Ti}} = 5.04. \] Our method of comparing abundances rather than \( f \)-values has the advantage of not being restricted to common lines. The \( f \)-values measured by Ellis (1976) (Sect. 4) lead to \[ \log e_{\text{Ti}} = 4.95. \] Thus we find basic consistency of all three sets, the Whaling et al. absolute scale being some 25% lower than the others.

In order to compare solar and meteoritic abundances one usually employs Si as the common reference element. However, regarding the desirable variety of solar lines and of reliable \( f \)-values, Ca appears preferable. The solar calcium abundance, \[ \log e_{\text{Ca}} = 6.36 \] (Holweger, 1972; Ayres, 1976) leads to a solar abundance ratio Ti/Ca = 3.8 \( \times \) 10\(^{-2}\) with an uncertainty of about 25 to 30%, depending on the judgement of the Ti \( f \)-value absolute scale. The meteoritic ratio (Mason, 1971) is Ti/Ca = 3.3 \( \times \) 10\(^{-2}\) in C1 chondrites and increases towards C2 and C3 (4.0 and 4.8 \( \times \) 10\(^{-2}\), respectively). Ordinary chondrites group around C3 (4.2 to 5.6 \( \times \) 10\(^{-2}\)), enstatite chondrites are somewhat higher (E4: 5.6 \( \times \) 10\(^{-2}\), E6: 5.4 \( \times \) 10\(^{-2}\)). Clearly chondrites contain the highly refractory elements Ti and Ca more or less in solar proportions, and there is evidence that in particular types C1 and C2 match solar matter best.

### 3. Comments on the Height Dependence of the Solar Velocity Field

Abundance studies offer a direct approach to separate small-scale motions (microturbulence) from the total velocity field. However, as pointed out by Garz et al. (1969) in the case of Fe i, it is difficult to distinguish between height-dependent and constant microturbulence. The present study corroborates this. Our solar model includes the microturbulence model tentatively considered also by Garz et al. (gaussian distribution, most probable velocity \( \xi_{\text{micro}} \) decreasing with height). We have recalculated all lines of Table 2, including the stronger ones, with constant \( \xi_{\text{micro}} = 1.0 \) km s\(^{-1}\). This leads to exactly the same mean abundance and standard deviation as before, demonstrating a lack of height discrimination that prevents at present any decision between height-dependent and constant microturbulence models. Improved height resolution may be possible by comparing low-excitation and ion lines. The critical point will be the \( f \)-values.

Contrary to this situation there is evidence that the total nonthermal velocity, \( \xi \), as determined from widths and centers of lines of different excitation, decreases throughout the layers \( \tau_{\text{5000}} \approx 1 \) to \( \approx 10^{-2} \) (Canfield and Beckers, 1976). A recent study by Ayres (1977) appears to conflict with that, indicating a constant \( \xi = 1.6 \) km s\(^{-1}\). However, the alleged height resolution in the lower photosphere (Fig. 3, l.c.) is fictitious: the height of formation of the emergent intensity of the local continuum should not be identified with the height where superimposed weak absorption lines originate (Elste, 1955; Sorli and Engvold, 1972; Müller et al., 1975).

### 4. Precision Oscillator Strengths as a Test for LTE

Ellis (1976) has measured \( f \)-values of 8 selected weak solar Ti i lines using the Oxford absorption furnace (Collins et al. 1976).

#### Table 2. Solar Ti i lines used for abundance analysis. Equivalent widths, \( W \), refer to disk center

<table>
<thead>
<tr>
<th>( \lambda ) (Å)</th>
<th>Mult. No.</th>
<th>E.P. (eV)</th>
<th>( W ) (mA)</th>
<th>( \log gf )</th>
<th>( \log e )</th>
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<tr>
<td>4656.47</td>
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<td>4681.91</td>
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<tr>
<td>4715.30</td>
<td>6</td>
<td>0.05</td>
<td>9</td>
<td>-2.75</td>
<td>5.12</td>
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<tr>
<td>5009.65</td>
<td>5</td>
<td>0.02</td>
<td>27</td>
<td>-2.17</td>
<td>5.06</td>
</tr>
<tr>
<td>5064.66</td>
<td>5</td>
<td>0.05</td>
<td>87</td>
<td>-0.88</td>
<td>5.12</td>
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<tr>
<td>5192.98</td>
<td>4</td>
<td>0.02</td>
<td>80</td>
<td>-1.04</td>
<td>5.07</td>
</tr>
<tr>
<td>5210.39</td>
<td>4</td>
<td>0.05</td>
<td>87</td>
<td>-0.86</td>
<td>5.08</td>
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<tr>
<td>5366.65</td>
<td>35</td>
<td>0.81</td>
<td>2.5</td>
<td>-2.49</td>
<td>4.99</td>
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<td>5389.18</td>
<td>35</td>
<td>0.81</td>
<td>5</td>
<td>-2.09</td>
<td>4.89</td>
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<tr>
<td>5866.45</td>
<td>72</td>
<td>1.06</td>
<td>44</td>
<td>-0.75</td>
<td>4.93</td>
</tr>
<tr>
<td>5880.27</td>
<td>71</td>
<td>1.05</td>
<td>6</td>
<td>-1.70</td>
<td>4.78</td>
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<tr>
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<td>72</td>
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<td>4.95</td>
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<td>5918.55</td>
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<td>-1.51</td>
<td>4.82</td>
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<td>7188.55</td>
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<tr>
<td>7357.74</td>
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<td>21</td>
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<td>4.96</td>
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<td>7364.11</td>
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<td>15</td>
<td>-1.13</td>
<td>4.91</td>
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</table>

#### Oscillator strengths of Kühne et al. (1977)

<table>
<thead>
<tr>
<th>( \lambda ) (Å)</th>
<th>Mult. No.</th>
<th>E.P. (eV)</th>
<th>( W ) (mA)</th>
<th>( \log gf )</th>
<th>( \log e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4715.30</td>
<td>6</td>
<td>0.05</td>
<td>8.8</td>
<td>-2.61</td>
<td>4.98</td>
</tr>
<tr>
<td>4926.16</td>
<td>39</td>
<td>0.82</td>
<td>5.1</td>
<td>-2.10</td>
<td>4.95</td>
</tr>
<tr>
<td>5009.65</td>
<td>5</td>
<td>0.02</td>
<td>26.8</td>
<td>-2.06</td>
<td>4.95</td>
</tr>
<tr>
<td>5045.45</td>
<td>38</td>
<td>0.85</td>
<td>8.0</td>
<td>-1.90</td>
<td>4.97</td>
</tr>
<tr>
<td>5295.78</td>
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<td>1.07</td>
<td>11.2</td>
<td>-1.49</td>
<td>4.91</td>
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<td>5460.51</td>
<td>3</td>
<td>0.05</td>
<td>8.2</td>
<td>-2.67</td>
<td>4.94</td>
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<tr>
<td>6064.63</td>
<td>69</td>
<td>1.05</td>
<td>7.8</td>
<td>-1.78</td>
<td>4.95</td>
</tr>
<tr>
<td>6126.22</td>
<td>69</td>
<td>1.07</td>
<td>19.1</td>
<td>-1.35</td>
<td>4.97</td>
</tr>
</tbody>
</table>

* Equivalent width also adopted from this source.

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et al., 1970). Apart from the quoted 0.08 dex uncertainty of the absolute scale the relative accuracy is claimed to be 0.03 dex.

We have checked this by computing solar abundances in the same way as in Section 2, adopting the equivalent widths given by Ellis. The remarkable small spread of abundances (Table 2) is in accordance with Ellis’ results and confirms the precision of the f-values. The mean value and standard deviation is log \( e_{\text{Ti}} = 4.95 \pm 0.02 \).

The excitation potential of the lower levels ranges from 0.02 to 1.07, that of the upper levels from 2.31 to 3.93 eV. A variety of different terms are involved. The small spread of the individual abundances computed under the assumption of LTE strongly indicates that any departures from LTE among these 10 different terms are smaller than about 5% or, if larger, are of equal magnitude (which seems rather improbable).

5. A Test for the Accuracy of Ti I STFD Oscillator Strengths of Kurucz and Peytremann (1975)

Kurucz and Peytremann have provided a very comprehensive list of theoretical f-values, most of them being based upon scaled Thomas-Fermi-Dirac (STFD) radial wavefunctions. As these authors emphasize, it is difficult to estimate errors in both absolute and relative scale on the base of their calculations alone, and they admit a factor of two for some unspecified ions. An independent test of these f-values will undoubtedly add to the usefulness of their work. As demonstrated in previous sections and in earlier work on calcium, a solar abundance study is well-suited for this purpose.

The log gf values following from Table 2 together with the log gf from Kurucz and Peytremann (1975) lead to Ti abundances plotted vs. equivalent width in Figure 1b. The isolated point is \( \lambda 5918 \); the tabulated log gf = −3.87 is clearly in error, the line behaving normally if the experimental f-value is used.

For comparison, the results obtained from the experimental f-values are shown in Figure 1a. Mean values and standard deviations for the 13 lines weaker than 60 mÅ are

\[ \log e_{\text{Ti}} = 4.94 \pm 0.10 \] (experimental f-values of Kühne et al., 1977);

\[ \log e_{\text{Ti}} = 5.09 \pm 0.32 \] (STFD f-values of Kurucz and Peytremann, 1975; \( \lambda 5918 \) excluded);

\[ \log e_{\text{Ti}} = 5.26 \pm 0.68 \] (as above, but \( \lambda 5918 \) included).

This demonstrates that even for complex spectra such as Ti I, STFD calculations can provide reasonable f-values if occasional drastic failures can be identified.

References

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