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Abstract:

Passing alpha particles in a Helias reactor (Major radius 22 m, plasma radius 1.8 m, magnetic field 5 T) are well confined if only Coulomb collisions perturb the particle orbits and neoclassical diffusion is the only transport process. In a turbulent plasma close to the beta limit particle orbits can become chaotic and enhanced losses of these particles may arise thus diminishing the alpha particle heating rate. The paper describes the guiding center orbits of passing particles in the magnetic field of the Helias reactor, where islands of the magnetic surfaces occur at the boundary. Drift surfaces also exhibit these islands, however, resonance surfaces with islands in drift surfaces can also exist inside the plasma. Numerically the rotational transform of the drift orbits has been computed verifying this feature. Using a mapping technique, the long term behaviour of the drift orbits is analysed taking into account the variation of energy due to plasma oscillations. This leads to two coupled area-preserving maps, which describe enhanced radial transport of particles satisfying the resonance condition. In phase space the anomalous transport occurs along the resonance lines of the Arnold web. The energy of the particles can vary in a chaotic manner if the amplitude of the oscillations is large enough, in model calculations it is shown that resonant alpha particles can be rapidly lost by Arnold diffusion. If this loss of alpha particles only occurs at the beta limit, it may help to stabilize the operation point against thermal instability by reducing the heating power. If drift waves contribute to alpha particles losses way below the beta limit, the Arnold diffusion, this anomalous loss is threatening the ignition in the Helias reactor.
1. Introduction

Energetic alpha-particles in a Helias reactor\textsuperscript{1} can be disturbed by magnetic and electric fluctuations, in particular by Alfvén waves, which are excited at the beta limit. The interaction between particles and waves often has a resonance character and can lead to a rapid loss of the alpha particles. If this loss occurs on a faster time scale than one slowing down time the heating power of the plasma will be reduced. The majority of the alpha particles are passing particles, they do not change sign of their toroidal velocity. Slowing by Coulomb interaction is the dominant collisional process and the radial collisional diffusion of these particles is negligibly small. Trapped particles are a minority, however their collisional diffusion is larger than the diffusion of passing particles. Also trapped particle losses can be enhanced by plasma oscillations, this issue will be addressed in separate paper; in the present paper attention will be focussed on the anomalous behaviour of passing particles.

The rotational transform in a Helias reactor ranges from $t = 0.82$ on the axis to $t = 1$ at the boundary. Due to the 5-fold symmesty of the configuration 5 magnetic islands exist at the boundary which are used for divertor action. Drift surfaces of passing particles will exhibit the same islands, however because of the magnetic drift the radial position of the drift islands will be shifted inward or outward. The sign depends on the sign of the parallel velocity of the particles.

In the following we consider a time-independent magnetic field, which has closed flux surfaces. This field is the unperturbed field of the Helias reactor without islands and stochasticity. The magnetic surfaces of the unperturbed case are described by $\psi = \text{const.}$ The lowest order field is disturbed by time-independent magnetic fields, which generate the natural islands at $\text{iota} = 5/n$, $n = 4,5,6,\ldots$, and $10/(n_1+n_2)$. The flux surfaces are used as radial co-ordinates and by introducing flux co-ordinates the magnetic field lines become straight lines.

Passing particles stay close to the magnetic surfaces. Electric drift and magnetic drift, however, modify the drift rotational transform and cause a difference between magnetic rotational transform and drift rotational transform. In particular, the poloidal magnetic drift in the finite beta field of the Helias is needed to improve the confinement of trapped and highly energetic alpha particles. On the other hand, this poloidal drift creates a large difference between magnetic transform and drift rotational transform of passing alpha particles with high energy. The origin of the poloidal drift is either a radial electric field or the magnetic drift arising in a finite beta equilibrium. In Fig. 1 the drift rotational transform in a Helias reactor is shown, the magnetic field has been computed self-consistently. Radial electric fields have been neglected. Magnetic and electric perturbations can create islands on resonant surfaces\textsuperscript{2} and since the five-fold symmetry of the Helias configuration is the origin of magnetic islands at $\text{iota} = 5/n$, it will be expected that drift surfaces also exhibit these islands, however at another position than the magnetic islands.

Alpha particles are born with an energy of 3.5 MeV corresponding to a velocity of $1.3 \times 10^7$ m/s. In a Helias reactor ($R = 22$m, $L=138$m) the toroidal transit time is $T = 2.9 \times 10^{-6}$ s, consequently electromagnetic oscillations with a frequency of $\nu = 3.4 \times 10^2$ kHz will be in resonance with passing particles.

\textsuperscript{1} C.D. Beidler et al., IAEA Conf. on Controlled Fusion, IAEA-F1-CN-69/FTP/01, Yokohama 1998

\textsuperscript{2} A.A. Shishkin, I.N. Sidorenko, H. Wobig, IPP-report IPP 2/340 April 1998
Fig. 1: Drift rotational transform of passing alpha particles in a Helias reactor. Black curve: rotational transform of the magnetic surfaces in the finite beta equilibrium (NEMEC code, E. Strumberger). Energy of alpha particles: 3.5 MeV. The pitch angle $\lambda$ is the ratio between parallel velocity and total velocity of the particle at the initial point. The sign of $\lambda$ indicates the parallel or anti-parallel motion of the particles.

Fig. 1 shows that the drift rotational transform appreciably differs from the magnetic transform. For this reason resonances, which have been avoided in the magnetic field, may occur in the drift orbits. Islands, which do not occur in the magnetic field, may exist in the drift orbits.

The change of the rotational transform can be understood by the following simple arguments. Let us assume that there is a radial gradient of the magnetic field leading to a poloidal drift. The modification of the rotational transform is roughly

$$\delta t = \frac{R v_p \rho_a}{r v_p} \frac{\delta B}{2r B}$$

Eq. 1.1

$r$ is the average radius of the magnetic surface and $\rho_a$ the gyroradius. The shift of the rotational transform depends on the energy of the particles, which implies that the island formation by symmetry breaking fields also depends on the energy of the particles. A further reason for modifying the rotational transform is the electric drift in radial direction. Let us assume that the thermal ions are electrically confined (ion root) which yields an electric drift in the order of

$$v_E = \frac{\rho_L}{r} v_{ia}$$

Eq. 1.2

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The shift of the rotational transform is in this case
\[ \delta t = \frac{R v_\perp p_\perp}{r v_n r} \]  
Eq. 1.3

The shift of the rotational transform depends on the sign of the parallel velocity. Although the magnetic surface has a resonant surface at the boundary \( i = 1 \) the resonant drift surface of some alpha particles is shifted to the interior of the plasma (see Fig. 2). Furthermore, close to plasma center a drift rational surface arises with \( i = 5/6 \), although the magnetic surface does not exhibit this rational surface. One may argue that the resonance and a possible stochastic behaviour of the orbits only affect particles with specific energy. However, since during the slowing down procedure alpha particles change their energy, all particles can arrive at this position and can get lost rapidly.

2. Basic equations

Let us consider a toroidal equilibrium with nested pressure surfaces. There may be islands and stochastic regions and a last magnetic surface surrounded by a stochastic sea. Flux surfaces do not exist everywhere and therefore the use of magnetic co-ordinates or Boozer co-ordinates may not be applicable everywhere. In this case the so-called canonical co-ordinate system proposed by Hazeltine and Meiss\(^4\) is more appropriate to describe particle orbits and to explore the general feature of these. This co-ordinate system has closed toroidal surfaces \( r = \text{const.} \), which are nearly tangential to magnetic field lines and they may coincide with magnetic surfaces where these exist. On these toroidal surfaces labelled by \( r \), two angular co-ordinates are chosen such as to eliminate the radial covariant components of the magnetic field and its vector potential. The co-ordinate system is defined by \( r, \theta, \phi \) with
\[ A = (0, A_\theta, A_\phi) \quad ; \quad B = (0, B_\theta, B_\phi) \]  
Eq. 2.1

It is always possible to annihilate the radial component of \( A \) by a gauge transformation. Eliminating the radial component of \( B \), however, requires a special choice of either the poloidal or the toroidal angular co-ordinate. The existence of such angular co-ordinate system, where the poloidal coordinate \( \theta \) has been replaced by a time dependent angular coordinate \( \eta \). Details are discussed in ref. 4.

In this coordinate system the Hamiltonian of the guiding center is
\[ H = \frac{B^2}{2mB_\phi^2} (p_\perp - qA_\perp)^2 + \mu B (p_\eta, p_\phi, \eta, \phi, t) + q \Phi (p_\eta, p_\phi, \eta, \phi, t) + p_\eta g \]  
Eq. 2.2

and the canonical momenta are given by
\[ p_\eta = \frac{\mu B_\eta}{B} + qA_\eta \]  
Eq. 2.3
\[ p_\phi = \frac{\mu B_\phi}{B} + qA_\phi \]  
Eq. 2.4

\( g \) is a given time-dependent function, which is determined by the coordinate transformation\(^5\). The vector potential of the magnetic field is the sum of three terms, which

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\(^5\) H. Wobig, D. Pfirsch, IPP report III/345, Sept. 1999
describe the lowest order field and the time-independent perturbations creating the natural islands and the time-dependent perturbation by the Alfvén modes.

\[ A_{\psi} = \chi(\psi) + \varepsilon A_{\psi}^1(\psi, \eta, \varphi) + \varepsilon A_{\psi}^2(\psi, \eta, \varphi, t) \quad \text{Eq. 2.5} \]

\[ A_{\eta} = \psi + \varepsilon A_{\eta}^1(\psi, \eta, \varphi) + \varepsilon A_{\eta}^2(\psi, \eta, \varphi, t) \quad \text{Eq. 2.6} \]

we have introduced the smallness parameter \( \varepsilon \), which will be used for expanding the canonical momenta. Inverting the Hamiltonian for passing particles yields the toroidal momentum as a new Hamiltonian

\[ p_{\psi} = qA_{\psi} + \sigma \sqrt{\frac{2mB^2}{B^2}(E - \mu B - q\Phi - p_{\psi}k)} = -K(E, t, p_{\eta}, \eta, \varphi) \quad \text{Eq. 2.7} \]

Since the poloidal magnetic field is small compared with the toroidal one we may approximate the poloidal canonical momentum by

\[ p_{\eta} = qA_{\eta} \quad \text{Eq. 2.8} \]

Together with eq. 2.6 we obtain

\[ q\psi = p_{\eta} + \varepsilon \psi_1(p_{\eta}, \eta, \varphi, t) + o(\varepsilon^2) \quad ; \quad \psi_1 = A_{\psi}^1(p_{\eta}, \eta, \varphi) + A_{\psi}^2(p_{\eta}, \eta, \varphi, t) \quad \text{Eq. 2.9} \]

This yields

\[ A_{\psi} = \chi(p_{\eta}) + \chi'(p_{\eta})\varepsilon \psi_1 + \varepsilon A_{\psi}^1(p_{\eta}, \eta, \varphi) + \varepsilon A_{\psi}^2(p_{\eta}, \eta, \varphi, t) + o(\varepsilon^2) \quad \text{Eq. 2.10} \]

The Hamiltonian \( K \) of circulating particles becomes

\[ -K = q\chi(p_{\eta}) + \varepsilon q\chi_1(p_{\eta}, \eta, \varphi, t) + \frac{mB^2}{B^2} \sqrt{\frac{2m}{E - \mu B - q\Phi}} \quad \text{Eq. 2.11} \]

where we have collected the perturbations in one term

\[ q\chi_1 = \chi'(p_{\eta})\psi_1 + A_{\psi}^1(p_{\eta}, \eta, \varphi) + A_{\psi}^2(p_{\eta}, \eta, \varphi, t) + o(\varepsilon) \quad \text{Eq. 2.12} \]

The first term describes the unperturbed orbits on the magnetic surfaces, in this approximation there is no difference between magnetic field lines and particle orbits. The second term is the distortion of the magnetic surface by time-independent and time-dependent magnetic perturbations. The third term causes the drift surfaces to differ from magnetic surfaces.

The magnetic field strength and the electrostatic potential are functions of the canonical variables \( p_{\eta}, \eta, E, t \), the independent variable is the toroidal angle \( \varphi \). The magnetic field strength has a small toroidal and poloidal variation

\[ B = B_0(p_{\eta}) + \delta B(p_{\eta}, \eta, \varphi) \quad \text{Eq. 2.13} \]

and the electric potential can be written as

\[ \Phi = \Phi_0(p_{\eta}) + \delta \Phi(p_{\eta}, \eta, \varphi, t) \quad \text{Eq. 2.14} \]

The last term describes the inhomogeneity of the electric potential and the time-dependent oscillations. Expanding the Hamiltonian with respect to \( \delta B \) and \( \delta \Phi \) leads to the lowest order Hamiltonian
\[-K_0 = q\chi(p) + \frac{mB_{\phi,0}}{B_0} \sqrt{\frac{2}{m} (E - \mu B_0 - q\Phi_0)} \tag{Eq. 2.15}\]

\[
K_1 = \frac{B_{\phi,0}}{B_0 \sqrt{\frac{2}{m} (E - \mu B_0 - q\Phi_0)}} (\mu \delta B + q \delta \Phi) \tag{Eq. 2.16}\]

The variation of \(B_\phi/B\) has been neglected. The perturbation is the sum of a time-independent and a time-dependent part.

\[
K_{1,1} = \frac{B_{\phi,0} \mu \delta B}{B_0 \sqrt{\frac{2}{m} (E - \mu B_0 - q\Phi_0)}} ; \quad K_{1,2} = \frac{B_{\phi,0} q \delta \Phi}{B_0 \sqrt{\frac{2}{m} (E - \mu B_0 - q\Phi_0)}} \tag{Eq. 2.17}\]

In the following we drop the index \(\eta\) and denote the poloidal momentum by \(p\). The lowest order Hamiltonian describes the motion in magnetic surfaces

\[
\frac{dp}{d\varphi} = -\frac{\partial K_0}{\partial \eta} = 0 ; \quad \frac{d\eta}{d\varphi} = \frac{\partial K_0}{\partial p} = t(p) \tag{Eq. 2.18}\]

The rotational transform of the particle is the sum of two terms: The first term is the rotational transform of the magnetic field \(u(\psi)\) and the second term is the result of magnetic drift and electric drift. Especially a lowest order electric field will modify the drift rotational transform.

\[
t = \frac{d\chi(p)}{dp} + \frac{\partial}{q \partial p} \left( \frac{mB_{\phi,0}}{B_0} \sqrt{\frac{2}{m} (E - \mu B_0 - q\Phi_0)} \right) \tag{Eq. 2.19}\]

If the electric field is large enough the drift rotational transform can be approximated by

\[
t = \frac{d\chi(p)}{dp} - \frac{B_{\phi,0}}{B_0} \frac{\partial \Phi_0}{\partial p} ; \quad u = \sqrt{\frac{2}{m} (E - \mu B_0 - q\Phi_0)} \tag{Eq. 2.20}\]

In case of highly energetic alpha particles the magnetic drift dominates and the drift rotational transform is

\[
t = \frac{d\chi(p)}{dp} - \frac{B_{\phi,0}}{qB_0} \frac{\mu \partial B_0}{\partial p} ; \quad u = \sqrt{\frac{2}{m} (E - \mu B_0 - q\Phi_0)} \tag{Eq. 2.21}\]

Taking into account the finite perturbation leads to the following set of canonical equations

\[
\frac{dp}{d\varphi} = -\frac{\partial K_{1,1}}{\partial \eta} - \frac{\partial K_{1,2}}{\partial \eta} ; \quad \frac{d\eta}{d\varphi} = t(p) + \frac{\partial K_1}{q \partial p} \tag{Eq. 2.22}\]

\[
\frac{dE}{d\varphi} = \frac{\partial K_{1,2}}{\partial \chi} ; \quad \frac{dt}{d\varphi} = T(p,E) - \frac{\partial K_1}{\partial E} \tag{Eq. 2.23}\]

\(T\) is the toroidal transit time of the circulating particles. This time is a decreasing function of the energy, since fast particles have a smaller toroidal transit time than slow ones. According to Liouville's theorem the motion in the four-dimensional phase space is
volume-conserving and can be described as a sequence of canonical transformations. The action generating function of the toroidal map, which maps the initial point \((p_0, \eta_0, E_0, t_0)\) onto \((p_1, \eta_1, E_1, t_1)\) has the form

\[
S = S(p_1, \eta_1, E_1, t_1) = p_0 \eta_0 + E_0 t_0 + K_0(E_1, p_1) + S_{1,1}(p_1, \eta_1, E_1) + S_{1,2}(p_1, \eta_1, E_1, t_1)
\]

Eq. 2.24

and the mapping equations are

\[
E_0 = E_1 + \frac{\partial S_{1,2}}{\partial \eta_0} ; \quad t_0 = t_1 + T(E_1, p_1) + \frac{\partial S_{1,1}}{\partial E_1} + \frac{\partial S_{1,2}}{\partial E_1} \quad \text{Eq. 2.25}
\]

\[
p_0 = p_1 + \frac{\partial S_{1,1}}{\partial \eta_0} + \frac{\partial S_{1,2}}{\partial \eta_0} ; \quad \eta_1 = \eta_0 + t_1 + \frac{\partial S_{1,1}}{\partial p_1} + \frac{\partial S_{1,2}}{\partial p_1} \quad \text{Eq. 2.26}
\]

The time-independent part \(S_{1,1}\) describes the deviation of drift surfaces caused by magnetic drifts and electric drifts. This model has been used to compute the drift surfaces on Fig. 2. The Fourier spectrum of Alfvén waves and of electrostatic drift waves in general is very rich and has many frequencies, which are not always multiples of each other. The perturbations \(S_{1,1}\) and \(S_{1,2}\) are periodic functions of the angles and of the time. We expand the perturbations in a Fourier series

\[
S_{1,1} = \sum_i a_i(p_1, E_1) \exp(i \eta_0) \quad \text{Eq. 2.27}
\]

and

\[
S_{1,2} = \sum_i b_i(p_1, E_1) \exp(i \eta_0 + n \omega_0) \quad \text{Eq. 2.28}
\]

The resonance condition becomes

\[
t_1(p, E) + n \omega T(p, E) = m = 0 \quad \text{Eq. 2.29}
\]

The resonance condition eq. 2.29 describes lines in the \(p, E\)-plane. For every integer \(l, n, m\) there is a line and all together these resonance lines build up the Arnold web\(^6\).

3. The Arnold web in a Helias configuration

The magnetic surfaces in a Helias reactor exhibit 5 islands at the edge and – if the 5/6 resonance exists in the center – also 6 islands close to the plasma center. Drift orbits with slightly higher transform than magnetic surfaces exhibit an inward shifted resonance surface and thus drift islands in side the main plasma. A model of such a drift surface is shown in the following Fig. 2.

As a model for a Helias configuration we approximate the drift rotational transform be a linear function in \(p\) and neglect an dependence on the energy. The toroidal transit time is approximated by a \(p\)-independent function

\[
t_1(p) = t_0 + t_1 p ; \quad n \omega T = \frac{n}{\sqrt{E}} \quad \text{Eq. 3.1}
\]

Taking more and more integers makes the web more and more dense.

\(^6\) A.J. Lichtenberg, M.A. Lieberman, *Regular and Stochastic Motion*, Springer Verlag 1983
Fig. 2: Model of drift orbits in a Helias reactor. There are islands at \( i = 5/6, \) 10/11 and \( i = 5/5. \) The number of islands is equal to the denominator of iota. The islands are shifted inward as compared with magnetic surfaces. \( i (0) = 0.82, \) \( i (1) = 1.05. \) The coefficients \( s_1 \) and \( s_2 \) are set to zero. \( a_5 = 0.02, \) \( a_6 = 0.005. \)

Fig. 3: Arnold web in the \( p_E \) plane. \( N=1, l=0, \pm 1, \pm 4, m=0, \pm 1, \pm 4. \)
In Fig. 3 only a few lines of the Arnold web are shown. In principle the Arnold web is a system of resonance lines on the energy surface in the 3-D action space, however, here the resonance condition depends only on two canonical variables \(p,E\) and the Arnold web can be described by its 2-dimensional projection into the \(p,E\)-plane.

The toroidal transit time scales inversely with the parallel velocity, which has been approximated by \(\sqrt{E}\). This is not quite correct, since there is a lower threshold of the energy, which separates passing particles from trapped particles. However, if the energy of the particles is large enough this approximation may be applicable.

4. Model of plasma perturbation

The generating function of the particles without perturbations is

\[
S_0 = p_0 \eta_0 + \int pf(p)dp + E_0 t_0 + 2 \sqrt{E_1}
\]

Eq. 3.2

which leads to the mapping equations of the unperturbed case

\[
E_0 = E_1 \quad ; \quad t_1 = t_0 + \frac{1}{\sqrt{E_1}}
\]

Eq. 3.3

and

\[
p_0 = p_1 \quad ; \quad \eta_0 = \eta_0 + t(p_1)
\]

Eq. 3.4

and the perturbation approximated by

\[
S_{1,1} = \frac{p_1}{2\pi \sqrt{E_1}} \sum_i a_i \cos \left(2\pi \eta_0 \right)
\]

Eq. 3.5

and

\[
S_{1,2} = p_1 \left[ \frac{s_1}{2\pi} \cos \left(2\pi (\eta_0 - t_0) \right) + \frac{s_2}{2\pi} \cos \left(2\pi (\eta_0 + t_0) \right) \right]
\]

Eq. 3.6

The derivatives used in the mapping equations (see eqs. 2.25 and 2.26)

\[
\frac{\partial S_{1,1}}{\partial \eta_0} = -\frac{p_1}{\sqrt{E_1}} \sum_i a_i \sin \left(2\pi \eta_0 \right)
\]

Eq. 3.7

\[
\frac{\partial S_{1,1}}{\partial p_1} = \frac{1}{2\pi \sqrt{E_1}} \sum_i a_i \cos \left(2\pi \eta_0 \right)
\]

Eq. 3.8

\[
\frac{\partial S_{1,1}}{\partial E_1} = -\frac{1}{4\pi E_1} \sum_i a_i \cos \left(2\pi \eta_0 \right)
\]

Eq. 3.9

\[
\frac{\partial S_{1,2}}{\partial p_1} = \left[ \frac{s_1}{2\pi} \cos \left(2\pi (\eta_0 - t_0) \right) + \frac{s_2}{2\pi} \cos \left(2\pi (\eta_0 + t_0) \right) \right]
\]

Eq. 3.10
\[ \frac{\partial s_{1,2}}{\partial \eta_0} = p_i \left[ s_1 \sin(2\pi(\eta_0 - t_0)) - s_2 \sin(2\pi(\eta_0 + t_0)) \right] \quad \text{Eq. 3.11} \]

\[ \frac{\partial s_{1,2}}{\partial \eta_0} = -p_i \left[ s_1 \sin(2\pi(\eta_0 - t_0)) + s_2 \sin(2\pi(\eta_0 + t_0)) \right] \quad \text{Eq. 3.12} \]

The perturbation grows linearly with the radial co-ordinate \( p_i \). This may not be quite realistic, since the perturbations tend to be localised to some region, however in this approximation the mapping equations can be solved explicitly without the need of Newton iteration for \( p_i \).

The first term of the perturbation eq. 3.5 describes the deviation of drift surfaces from magnetic surfaces, the term scales inversely with the square root of the energy. The takes into account that slow particles are more strongly affected by perturbations than fast ones. In order to model drift surfaces in the Helias reactor the two harmonics with \( l=5 \) and \( l=6 \) are retained. The \( l=5 \) creates the 5 islands at the boundary and the \( l=6 \) mode the 6 islands at \( t=5/6 \) in the plasma interior. The results of the \( p,\eta \)-map are transferred to polar coordinates using \( x = \sqrt{p \cos \eta}, \ y = \sqrt{p \sin \eta} \).

**Fig. 4**: Diffusion of particles caused by resonant interaction with the waves. The particles with \( E = 1 \) start close to the separatrix of the island. \( s_1 = 0.004, \ s_2 = 0 \). The figure displays a 90°-sector of the drift surface. 20000 toroidal transits.
**Fig. 5**: Chaotic variation of the energy vs time. 10 particles, 20000 toroidal transits.

**Fig. 6**: Particle diffusion projected into the Arnold web. $s_1 = 0.004$, $s_2 = 0$. 
Fig. 7: Diffusion in the Arnold web. $s_1 = 0.0, s_2 = 0.004$

Fig. 8: Diffusion in the Arnold web. $s_1 = 0.004, s_2 = 0.004$
Fig. 9: 10 particles, 20000 transits. The particles start on a drift surface without islands. (line 40). In Fig. 2 this line is colored in black (thick line).

Fig. 10: Diffusion on the Arnold web, 10 particles, 20000 transits. The amplitude of the waves has been increased to $s_1 = 0.01$, $s_2 = 0.01$, $E = 1$. 
Fig. 11: Diffusion in $\tau=1$-island. 20000 transits, $s_1 = 0.01$, $s_2 = 0.01$, $E = 1$.

Fig. 12: Diffusion in $\tau=1$-island. 40000 transits, $s_1 = 0.01$, $s_2 = 0.01$, $E = 1$. 
Fig. 13: 10 particles, 40000 transits, start on KAM-surface (unperturbed drift surface, line 40). E=1.

Fig. 14: Diffusion on the Arnold web, 10 particles, 20000 transits. The amplitude of the waves has been increased to $s_1 = 0.01$, $s_2 = 0.01$, $E = 0.85$. 
Fig. 15: 10 particles, 40000 transits, start on KAM-surface (unperturbed drift surface, line 60), $E=0.85$.

Fig. 16: 30 particles, 40000 transits, start on KAM-surface (unperturbed drift surface, line 60), $E=0.90$. 
5. Conclusions

The present analysis has been restricted to a simple model of drift surfaces in a Helias reactor, however, it exhibits the basic features of resonant diffusion caused by particle-wave interaction. The diffusion takes place on the Arnold web and is energy selective. The model of the time-dependent perturbation consists of two waves, a forward wave (phase = $\eta_0 - t_0$) and a backward wave with the phase = $\eta_0 + t_0$. In the low shear configuration of the Helias reactor only one important resonant surface exists. This is a drift surface with $\tau = 1$. These drift surfaces are shifted inwards and exhibit the same island structure as the magnetic surfaces. Drift surfaces of alpha particles further inside are nested and without collision the particles do not experience a strong radial transport. Under the influence of time-dependent perturbations particles starting in the $\tau=1$-islands or on the separatrix of the island are strongly perturbed and become chaotic. There is a rapid diffusion across the island (see Fig. 2). The diffusion follows the pattern of the Arnold web (see Fig. 6) and depending on the amplitudes of the forward or backward waves, the diffusion on the Arnold web behaves quite differently (Figs. 6, 7, 8). If the particle starts outside the Arnold web. The radial diffusion of the particles is rather small (Fig. 9). Raising the amplitude of the waves leads to a faster radial diffusion (Figs. 10, 11, 12). Also in this case a start off the resonance lines shows only a small radial transport (Fig. 13).

Fig. 14 demonstrates that particles can also start outside the island region where the unperturbed drift surfaces are nested and still undergo strong radial transport if the resonance condition is met. The diffusion in spatial coordinates is displayed in Fig. 15. If these particles start closer to the $\tau=1$-island they can diffuse into the island and get rapidly lost to the outer boundary (Fig. 16).

Although computations are using model perturbations and also a model for the unperturbed drift surfaces it is believed that the generic behaviour of passing alpha particles in the Helias reactor is represented. Particles starting outside the Arnold web are only negligibly affected by time-dependent oscillations. Arnold diffusion, however, becomes strong if the particles start close to $\tau=1$-islands or further inside the plasma on the Arnold web. The basic mechanism of this enhanced diffusion is the variation of the energy by the plasma oscillation. The time variation of the energy as displayed in Fig. 5 becomes chaotic and the resulting motion in the p,\eta-plane shows strong radial diffusion. It is remarkable that the energy stays bounded although the variation is chaotic. Whether this is a general feature or a special result in the present parameter space remains to be investigated.

The resonant alpha particles change energy also by the slowing down process. They can diffuse towards the Arnold web if they have started outside and they can fall off the resonance lines due to collisions. The loss of alpha particles is the combined result of Arnold diffusion and collisional diffusion. The relation to neoclassical transport of alpha-particles is evident since particle orbits are the characteristics of the drift-kinetic equation. The neoclassical theory corresponding to the previous calculations would retain time-dependent electric and magnetic fields and investigate the combined effect of particle-wave interaction and collisions.