HYPOTHESES ON L-H TRANSITION

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Abstract

Two hypotheses in connection with L-H transition are examined. The first is that the transition is triggered by the radial temperature gradient in a cold edge region exceeding a threshold. A number of observations relating to the occurrence of the H-phase can be understood qualitatively from this hypothesis. A possible theoretical basis in terms of stabilization of drift waves is found. The second hypothesis is that transport at the edge region in the H-phase is governed by neoclassical theory for electrons and ions. Exploration of the consequence on the temporal behavior of various signals indicates that while energy transport appears consistent with the predictions, particle transport appears much faster than neoclassical theory from the limited diagnostic information available.

I. Introduction

It is now well documented (1) that the L-H transition in ASDEX is accompanied by a sudden improvement of particle and energy confinement in an edge region of a few centimeters in thickness inside the separatrix. In this connection, several questions naturally arise: Is edge improvement alone sufficient to explain the improvement of overall confinement in the H-phase without invoking additional change of transport mechanism in the plasma interior? What are the transport coefficients in the edge region during the H-phase? What is the critical edge condition which triggers the transition?

In this report we examine the consequence of two hypotheses. The first is that the L-H transition is triggered by the radial temperature gradient in a cold enough region, and thus near the edge, exceeding a critical value. The second is that transport in the edge region in the H-phase is governed by neoclassical theory for both electrons and ions.

It is found that with the first hypothesis, a number of observations relating to the conditions of the occurrence of the H-phase, such as power threshold, density threshold etc., can be qualitatively understood. A

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possible theoretical basis in terms of stabilization of drift waves is also uncovered. The second hypothesis meets with mixed success when its predictions on the temporal behavior of various signals are compared with observations. However, through its examination it has come to light that the observed overall confinement improvement can indeed be understood in terms of an edge barrier alone.

II. Critical Edge Conditions at Transition

It has been suggested that the occurrence of high edge temperature is what leads to the transition into the H-phase \((2)\). However in the edge region of a few cm inside the separatrix where the transport barrier is built up, temperature ranges from a few ten to hundreds of electron volts, and it is not clear exactly where the high temperature which triggers a transition is to be located. In this section we put forward an alternative hypothesis which states that the L-H transition occurs when the radial temperature gradient in a cold enough region near the edge exceeds a critical value. Two parameters thus occur in this hypothesis: a width \(\Delta\) where the temperature is low, which might for instance be connected with a plasma collisionality condition such as \(\nu_\ast \gtrsim 1\), and a critical temperature gradient \((dT/dr)_c\). We shall not distinguish between electron and ion temperatures. In this view, the temperature at the separatrix is held at small values due to the rapid parallel heat conduction onto the divertor plates, so that continuous heating of the plasma interior by neutral beams or other means increases the edge temperature gradient until a threshold is crossed, when the transport coefficients in a cold region of width \(\Delta\) become small, perhaps as a result of a decrease or complete disappearance of turbulence. \(\Delta\) is a few cm and \((dT/dr)_c\) is around 100 eV/cm.

It will be shown that a number of observations relating to the conditions of the occurrence of the transition can be understood qualitatively in terms of this hypothesis.

1) Power threshold. For a given power \(P\), the maximum temperature gradient which can be supported at the edge during the L-phase of the discharge is given by the equation

\[
\frac{P}{4\pi^2 \text{Ra}} = \kappa_L \frac{dT}{dr}
\]

where \(\kappa_L\) is the edge thermal conductivity. A transition cannot occur if this maximum is below the critical gradient. Thus the power which satisfies
this equation with a critical temperature gradient is the power threshold for the occurrence of H-mode. We cannot verify this relation since we cannot measure $\kappa_L$ and $dT/dr$ accurately enough at transition. But crude estimates of these quantities can easily produce powers in the megawatt range, consistent with observations.

(2) Density thresholds. Writing $\kappa_L = n\chi_L$, and postulating in addition that $\chi_L$ depends only weakly on the density $n$, the power threshold becomes an increasing function of density, which means that for a given available power, density has to stay below a critical level to allow transition. When density becomes too small, on the other hand, penetration of neutral atoms increases, the accompanying increase of edge region radiation can prevent the building up towards the critical temperature gradient and so prevent transition.

(3) Unfavorable influence of limiter. Increased edge radiation from sputtered limiter material can also prevent transition for the same reason. It is noted, however, that H-mode is not absolutely impossible with limiters according to our hypothesis. One can conceive of introducing into the plasma suitable material which is strongly radiative only in an extremely thin layer near the limiter minor radius, which serves the purpose of maintaining the limiter at low temperature, reduces sputtering, while not causing too much edge radiation by itself.

(4) Isotope effect. Since deuterium plasma is better confined than hydrogen plasma, it has presumably smaller edge thermal conductivity also. This means lower power threshold as is observed to be the case.

(5) Transition time after NBI. The elevation of edge temperature gradient after neutral beam injection takes time. This time could be expected to become shorter the more the power is. This is indeed the case experimentally.

We turn next to some speculations on the theoretical basis of the hypothesis of critical temperature gradient.

As the pressure gradient at the edge steepens, the plasma digs deeper and deeper magnetic well until the well becomes absolute: the magnetic field
strength increases in all directions which go out radially from an isobaric surface. At a point on the outboard side of the toroid, the sum of kinetic and magnetic pressures are held in place by the tension of the magnetic field at equilibrium. A magnetic well at that point can only arise when the pressure force overcomes the tension, which occurs when

\[ \frac{dp}{dr} > \frac{B^2}{\mu_0 R} \]

It has been suggested that this is responsible for the improved confinement at the edge during the H-phase (3). The condition can be rewritten as the pressure gradient length \((1/p \, dp/dr)^{-1}\) being less than \(2\Omega R\) when \(\Omega\) refers to the edge plasma. For the reasonable choice \(n = 10^{13} \text{ cm}^{-3}\), \(T = 100 \text{ eV}\). \(B = 2.2 \text{ T}\) at transition, this length is only 0.014 cm for ASDEX \((R = 165 \text{ cm})\). The pressure gradient length at transition is certainly much longer, so that this mechanism cannot be the trigger.

Drift waves are commonly thought to cause anomalous transport in tokamaks. At low enough temperature when \(v_{*e} > 1\), trapped particle effects become unimportant and the relevant kinds are either collisional or collisionless drift waves depending on whether \(v_{ei} > k_n v_e\). The latter is stabilized when \(\eta_e = d\ln T_e/d\ln n\) and/or \(\eta_i = d\ln T_i/d\ln n\) exceed values of order unity (4). The stabilization is made easier by the presence of shear. However, experimental measurements of \(\eta_e\) and \(\eta_i\) at the edge are not precise enough to test out this possibility, which then could be regarded as a candidate still for the mechanism of L-H transition.

III. Temporal Consequences of an Edge Barrier

Convincing evidence has been presented that the L-H transition is accompanied by a sudden improvement in both particle and heat confinement in an edge region of a few cm inside the separatrix. While the improvement appears rather substantial, it has not been very well quantified. If the improvement is complete so that turbulence ceases to exist, neoclassical transport theory for both electrons and ions is expected to apply. If we assume that whatever mechanism that causes anomalous transport (certainly of the electrons) in the L-phase is still present in the plasma interior during the H-phase, it should then be possible to predict the temporal behavior of various signals inside and outside of the thin neoclassical layer and compare them with observations.
We begin by estimating the magnitudes of the expected neoclassical fluxes. According to Ref. 5, the particle flux, electron energy flux and ion energy flux can be written as

\[
\Gamma = -n_e \sqrt{\varepsilon} \frac{\rho_{e\theta}}{T_e} (K_{11} A_{1e} + K_{12} \frac{T_e'}{T_e}) - K_{13} n_e \sqrt{\varepsilon} \frac{E_n}{B_\theta} \\
q_e + \frac{5}{2} n_e \sqrt{\varepsilon} \frac{\rho_{e\theta}}{T_e} (K_{12} A_{1e} + K_{22} \frac{T_e'}{T_e}) - K_{23} n_e T_e \sqrt{\varepsilon} \frac{E_n}{B_\theta} \\
q_i = -K_2 n_i \sqrt{\varepsilon} \frac{\rho_{i\theta}}{T_i} \frac{dT_e}{dr}
\]

where

\[
A_{1e} = \frac{p_{e}}{p_{e}} - \frac{5}{2} \frac{T_e'}{T_e} + \frac{T_i'}{T_i} \left( \frac{p_{i}}{p_{i}} - K_i \frac{T_i}{T_i} \right)
\]

and the \(K\)'s are functions of collisionality parameters \(\nu_{*e}, \nu_{*i}\) as well as \(\varepsilon\). These formulas are derived under the assumption that poloidal gyroradius is much smaller than plasma gradient lengths, which, while satisfied by the electrons, is in fact somewhat marginal for ions because of the steep gradients which the edge plasma can eventually attain. When this happens, ion motion rapidly mixes the different flux surfaces and the existence of a well-defined temperature on each flux surface can no longer be maintained. These formulas are therefore taken only to provide rough estimates under the circumstances. We now make the following choice of parameters, which is typical of ASDEX operation in the H-phase:

\[
B = 2.2 \, \text{T}, \quad R = 1.65 \, \text{m}, \quad a = 0.4 \, \text{m}, \quad I = 0.35 \, \text{MA}, \quad q = 3, \quad V = 0.3 \, \text{Volts}, \quad n = 10^{13} \, \text{cm}^{-3}, \quad T = 300 \, \text{eV}, \quad 1/T \frac{dT}{dr} = 1/3 \, \text{cm}^{-1} \text{in D plasma. We shall further assume that } \Delta = 5 \, \text{cm}. \text{ This gives}
\]

\[
P = -D \frac{dn}{dr} + Vn
\]

with \(D = 33 \, \text{cm}^2/\text{sec}\) and \(V = -3 \, \text{cm/sec}\).

and

\[
q_i = -n_i \chi_i \frac{dT_i}{dr}
\]

with \(\chi_i = 520 \, \text{cm}^2/\text{sec}\). The small pinch velocity in the expression for the particle flux is the result of a large cancellation between the Ware pinch (7.8 cm/s) and an anti-pinch due to temperature gradient (4.6 cm/s). Such a
cancellation persists so long as the parameters stay close to the chosen values. The electron energy flux is much smaller than ion heat flux.

Electron heat transport is thus effectively stopped in the thin layer, and the retained power then raises the core electron temperature. The electrons then impart their energy to ions through collisional exchange, which is conducted to the outside through the thin layer in the ion channel. In the following we shall not distinguish between the two channels, and approximately describe the above process by ascribing $\chi_i$ to the thermal diffusivity for the total energy. We shall also neglect the pinch term in $\Gamma$, so that both particle and energy transport are described by equations of the same type.

We now introduce a simple model to describe the dynamics of pro-transition. The tokamak plasma is represented by two plane slabs (Fig. 1), with the one extending from $x = 0$ to $\Delta$ representing the edge, and the one from $x = \Delta$ to a the core. Neutral beam heating is mimicked by a time independent heat flux $Q$ entering the plasma at $x = a$, and beam fuelling by a corresponding flux $\Gamma$. Other particle sources are not considered. At $x = 0$, both temperature and density are held to small values by good parallel conduction and convection into the divertor plate, and are thus taken to be zero. Before transition, transport in both regions is described by a uniform diffusion coefficient $D_L$ and energy diffusivity $\chi_L$, with no convection. Transition occurs at $t = 0$, when these coefficients in the thin slab become $D$ and $\chi_i$ in the neoclassical theory. For simplicity, energy and particle transport will be decoupled by assuming in the energy transport equation that density is uniform and given by $\bar{n}$.

Consider first energy transport. A steady state temperature distribution in the L-phase given by

$$T_L(x) = \frac{Q}{3n\chi_L} x$$

is expected to be established in an L-phase confinement time $T_L = a^2/\chi_L$, while the H-phase steady state (Fig. 2)

$$T_H(x) = \begin{cases} \frac{Q}{3n\chi_i} x & 0 < x < \Delta \\ \frac{Q}{3n\chi_i} \Delta + \frac{Q}{3n} \chi_L (x-\Delta) & \Delta < x < a \end{cases}$$
will be established in the H-phase confinement time $\tau_H$, which we seek to determine. An inkling of what it might be can be obtained by taking the ratio of total energy in the steady state H-phase to the steady state L-phase. Under the approximation

$$\frac{X_i}{X_L} \ll \frac{\Delta}{a} \ll 1$$

(1)

which is reasonable for $X_i \sim 500$ cm/sec, $X_L \sim 10^4$ cm$^2$/sec, $\Delta = 5$ and $a = 40$, but is by no means essential as it only simplifies the algebra, this gives $2 \frac{X_L}{X_i} \frac{\Delta}{a}$, which should also be approximately the ratio $\tau_H/\tau_L$.

Thus we expect $\tau_H \sim \frac{\Delta a}{X_i}$.

A more reliable calculation involves solving the time dependent problem after the transition, assuming some initial condition. If this initial condition is close to the steady state solution for the L-phase, we expect at transition a sudden decrease by a factor $X_i/X_L$ for the heat flux leaking to the outside. This is the first prediction. We can solve for the subsequent evolution in time by using eigenfunction expansion method, which gives

$$T(x,t) = T_0(x) - \sum a \lambda_n(x) e^{-\lambda nt}$$

where the eigenvalues are given by the solution of

$$\tan \sqrt{\frac{\lambda_n}{\chi_i}} \tan \sqrt{\frac{\lambda_n}{\chi_L}} = \sqrt{\frac{X_i}{X_L}}$$

It turns out that under the approximation (1), the replacement of the tangent functions by their arguments is justified, leading to

$$\lambda_o = \frac{X_i}{a \Delta}$$

for the longest characteristic time, in agreement with our previous estimate.
According to this view, it takes a long time to reach the H-phase steady state if the phase remains quiescent (i.e. free from ELM's, which are presumably MHD phenomenon outside the scope of this work). The establishment of this steady state in reality is prevented by the occurrence of increasing core radiation, leading finally to the termination of the H-phase. It is not clear how the time scale $\lambda_o^{-1}$ is experimentally discernable under the circumstance. We note that immediately after the transition, many physical quantities take on small values relative to their final ones, and their time dependence is not governed simply by $\lambda_o$. In this connection, it is useful to introduce the doubling time $t_2$, which is the time it takes for a quantity to reach twice its value immediately after transition. Even though $t_2$ depends on the initial condition in a complicated manner, we argue that the equation

$$\xi(t) = \xi_\infty + (\xi_0 - \xi_\infty) e^{-\lambda_o t}$$

where $\xi_0$, $\xi_\infty$ are the initial and steady state values respectively, would yield an upper bound for $t_2$. In a typical situation where $\xi_\infty \gg \xi_0$, we thus obtain

$$t_2 = \frac{2 \xi_0}{\xi_\infty} \frac{1}{\lambda_o}$$

This formula is now applied to three quantities: the total energy, the temperature at $x = \Delta$ where the two slabs join, and the energy outflux at $x = 0$, giving the respective values $\frac{1}{2} \frac{a^2}{\chi_L}$, $\frac{1}{2} \frac{\Delta}{a}$, $\frac{1}{2} \frac{a^2}{\chi_L}$ and $\frac{1}{2} \frac{\Delta}{a} \frac{a^2}{\chi_L}$.

Below we summarize our findings and compare them with experimental observations.

(A) At transition, heat outflux suddenly decreases by a factor $\chi_\perp/\chi_L$. We do not know the value of $\chi_L$ near the edge very well although it can be measured in principle by simultaneously measuring the temperature gradient and the heat outflux right before transition. Taking as an estimate (6) $\chi_L = 10^4$ cm$^2$/s and the estimate of $\chi_\perp$ given before, we arrive at a reduction factor of 0.05, which considering the large uncertainty involved, is consistent with observation from power flow in the divertor (1).

(B) The total energy doubles itself after transition in somewhat less than an L-phase confinement time, which can be estimated to be 30 msec (7).
This appears to be only somewhat shorter than the 50 msec estimated by
the diamagnetic loop in reference 7. The situation could be improved
when core radiation is taken into account.

(C) The temperature at the core-edge interface doubles itself in less than
$\Delta/a$ times the L-phase confinement time. This corresponds to some 1.5
msec. Such a rapid variation is not supported by observations in ASDEX
as there is no ECE chord viewing this close to the edge. On the other
hand, PDX group did report such rapid variation (8).

(D) The heat outflux, after plunging at transition to a small value, would
climb back by a factor of two within the same short time as the edge
temperature in (C). This appears to be in gross disagreement with ASDEX
observation of heat flux and radiation in the divertor, which, after
dropping back to the Ohmic level from the elevated level during the L-
phase, remains there for some 100 msec. Inclusion of increasing radi-
ation loss is not likely to change the picture much in view of the short
predicted time. However, the failure to account for more than 40 % of
the input power during the H-phase (9) might tip the balance in favor of
the theory if the missing power turns out to go somewhere outside the
plasma.

The analysis of particle transport is exactly parallel to that of energy.
However, because of the uncertainty in the mechanism of fuelling, such as
the relative contribution of gas puff and neutral atoms returning from the
divertor chamber, particle confinement time in ASDEX, and thus the average
diffusion coefficient, are less accurately known than energy confinement
time. It is observed that line averaged density doubles itself after tran-
sition in 100 msec (1). According to our theory, this should be approximately
the L-phase confinement time. An averaged diffusion coefficient of 4,000
cm$^2$/sec can thus be inferred. Dividing this into 33 cm$^2$/sec, the estimated
neoclassical value, the theory also predicts that the particle flux at
transition falls to 0.8 % of the pre-transition value. If we take the atom
flux emerging from the neutralizer plate as an indicator of particle out-
flux, its decrease at transition is only down to 10 %, which is much less a
decrease than theory predicts. Allowing for the fact that fuelling by
ionization of edge neutrals is present during the H-phase, we might expect
the density doubling time be lengthened in its absence. This implies a
smaller L-phase diffusion coefficient and hence a less precipitous drop in
particle outflux at transition. However, it should be noted that beam fuelling alone falls short of accounting for the observed rate of density increase by less than a factor of 2–3, if at all. It is therefore hard to imagine such large changes brought about by the edge neutrals as to bring the theoretical outflux decrease in line with the decrease in reflected atomflux from the neutralizer plate.

We can summarize this section by saying that the barrier energy transport could be consistent with neoclassical predictions, with the provision that the power unaccounted for in the H-phase go somewhere outside the plasma. The latter provision is in fact crucial to barrier models of any diffusivity. The agreement would be equally good for any theory giving heat diffusivity of comparable value. However, the limited data on particle transport do not support the much smaller value of diffusion coefficient relative to energy diffusivity which neoclassical theory demands. To better test out this aspect of neoclassical theory, reliable measurement of particle outflux is essential.

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Fig. 1  The two-slab model
Fig. 2  Steady State Temperature in the L and H phases