Q PROFILE AS DETERMINED BY ECE

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Abstract

The current density distribution, and hence the $q$ profile, remains an important unmeasured function of plasma radius. Only the total plasma current, and, in the case of sawtoothing discharges, the approximate axial current density are known. By using standard models of plasma resistivity, a current density profile can be determined by the measured electron temperature profile.

A technique whereby the ECE data is used in this manner is described, together with the necessary assumptions required. Results are presented together with suggestions and warnings regarding their interpretation.
1. Introduction

A simple program has been developed to generate ASDEX current density profiles from electron cyclotron emission (ECE) measurements. These profiles can be used to determine the location of rational flux surfaces, the internal inductance, and the axial value of rotational transform, \( q_0 \). The precision of this method appears quite good, although the accuracy depends upon assumptions made in the modelling, and systematic errors inherent to the ECE measurements.

The required input data is read from the shot file, and includes the ECE measurements, the density, current, toroidal and vertical fields, and the loop voltage. All data is averaged over a 10 msec interval. Because of the chopper employed by the ECE system, certain times will be automatically altered to avoid inclusion of a chopper spike. Since the ECE diagnostic can provide only 4 spatial temperature points per discharge, the ability to include data from up to 10 shots is allowed. When multiple shots are summed, a 10% variation in input (such as density, current, or field) will be flagged, indicating conflicting data.

2. Description of Technique

By fitting an analytic function to the measured temperature data, the spatial temperature profile is described by the free parameters of the fitted function, and quantities such as the internal inductance and rotational transform become functions of these parameters. It is assumed that:

\[
T(r) = T_0 \left[ 1 - \left( \frac{S(r)}{a} \right)^2 \right]^{\alpha} \quad S(r) = r + \Delta \left[ 1 - \left( \frac{\pi}{a} \right)^2 \right]
\]

where \( S(r) \) is, to first order in inverse aspect ratio, the centre of circular flux surfaces of radius \( r \). The shift of the innermost surface is determined by the equilibrium control field:
\[ B_v = \frac{\mu_0 I}{4\pi R} \left[ \ln \frac{8R}{a} + \delta_{pol} + \frac{1i}{2} - \frac{3}{2} \right] \]

\[ \Delta = \frac{2\pi a^2 B_v}{\mu_0 I} + \frac{a^2}{2R} - \frac{a^2}{2R} \ln \frac{8R}{a} \]

where \(B_v, I\) are the vertical field and plasma current; \(a, R\) are the minor and major radii. It is assumed that rapid parallel heat conduction makes flux surfaces isothermal, and that the outer surface is centered. The fit is produced by iterative minimisation of the mean square deviation over a progressively restricted parameter range. These parameters are the central temperature \(T_0(r=\Delta)\), and the parabola power \(\alpha\).

The current density profile is determined by assuming classical resistivity together with a spatially uniform induced loop voltage. Classical resistivity allows momentum transfer from electrons only through coulomb collisions with single ions, hence collective effects and trapped electron orbits are not considered. The ion species may be described by \(Z_{eff}\), allowing for impurities, but \(Z_{eff}\) will be assumed constant in space. A uniform loop voltage requires the complete penetration of magnetic flux, allowing no expulsion of flux by sawteeth or skin effects. In the framework of this model:

\[ J(r) = \frac{(3\alpha + 2) I}{2\pi a^2} \left[ 1 - \left( \frac{S(r)}{a} \right)^2 \right] \frac{3\alpha}{2} \]

where the integrated current density has been normalised to the measured total current \(I\).

A more accurate treatment would reflect the fact that the current density is not a flux function. In the above treatment this is not the case.

The central value of \(q\) becomes:

\[ q_0 = \frac{4\pi B a^2}{\mu_0 I R (3\alpha + 2)} \]

where \(B\) is the central toroidal magnetic field. The radial dependence of \(q\) is given by:
\[ q(S(r)) = \frac{q_0 S(r)^2 (3\alpha + 2)}{2a^2 \left[ 1 - \left(1 - \frac{S(r)}{a}\right)^2 \right]^{\frac{3\alpha + 2}{2}}} \]

from which the location of rational q surfaces is derived. The internal inductance is reduced to the integral expression:

\[ LI = 2 \int_0^a \frac{dr}{r} \left[ 1 - \left(1 - \left(\frac{r}{a}\right)^2\right)^{\frac{3\alpha + 2}{2}} \right]^2 \]

which to within a few percent can be approximated by:

\[ LI = 1.08 + 0.54 \ln \alpha \]

A \( Z_{eff} \) can be determined from the measured loop voltage, \( V_L \), by comparing the measured temperature to the Spitzer results for a \( Z = 1 \) plasma:

\[
\left( \frac{T_{measured}}{T_{Spitzer}(B, V_L, q_0)} \right)^{3/2} = \frac{0.914 \ Z_{eff}}{1.077 + \ Z_{eff}} + 0.58 \ Z_{eff}
\]

Alternately, the information contained in \( Z_{eff} \) can be rendered in \( V_{calc} \), the loop voltage predicted by the measured temperature assuming \( Z_{eff} = 1 \).

3. Example Results

Example results of this procedure are shown in fig. (1) where data from similar shots at two times corresponding to before and during neutral injection (NI) are displayed. The computed current profile is drawn together with the measured temperature data scaled according to \( J(r) \propto T(r)^{3/2} \).

The fit parameters are written below the profile. 'Profile centre' refers to \( \Delta \), 'Central temperature' to \( T_c \) and 'Parabola power' to \( \alpha \). The calculated q profile is plotted and the radial (inside and outside
along the toroidal midplane) locations of the $q = 1$ and $q = 2$ surfaces are indicated. Plasma parameters are noted along with the calculated $Z_{eff}$ and internal inductance. It is often observed that during NI the calculated $Z_{eff}$ is less than unity. Since the measured loop voltage becomes quite small during this period, it is not clear if some form of current drive or merely measurement error produces this result.

The data points do not justify the use of any particular form of the fitted profile. For example, a gaussian function has been used with similar result. As more data is added to the fit, the necessity of the first order Shrafanov shift becomes manifest. In fig. (1) where data from the inner and outer plasma halves is used, a profile asymmetry is apparent, particularly during the NI phase. The inclusion of the Shrafanov shift in the fit means that only data from one profile half is strictly needed.

The time history of $q_0$ during current rise is shown in fig. (2). In this example small disruptions occur as $q_{lim}$ passes through integer values, so convective transport of flux possibly allows the assumption of a spatially uniform loop voltage. The minimum value of $q_0$ ($q_0 < 1$) is achieved at the onset of sawtooth disruptions, then remains constant during the current plateau. Fig. (3) shows the evolution of $q_0$ during current fall. Again, inductive effects have been ignored. Cessation of sawtooth disruptions is accompanied by an increase in $q_0$ above $q_0 = 1$. This correlation of $q_0$ with sawteeth is the only available check of the accuracy of the program, since $q_0 < 1$ is a recognised prerequisite for sawteeth.

The behaviour of the calculated $q_0$ during NI is shown in fig. (4) for the case of an H-type discharge. The increase in $q_0$ results from the enhanced broadening of the pressure profile. This calculated $q_0$ is the value in the ohmic case needed to reproduce the situation during NI. It is not clear to what extent the actual $q_0$ changes during NI.
4. Caveat

Because of a present difficulty in the frequency dependent calibration of the ECE diagnostic, a systematic error is introduced through the fitting process. This means that relative comparisons of results should be made between data having the same systematic error - usually by taking data from the same spatial locations (since the frequency is governed through the magnetic field). For example, fig. (5) shows the results of fig. (1) where the two profile halves are considered separately. The deviation in calculated $q_0$ results from the forementioned systematic error, and not from differing discharge condition. Thus caution should be exercised when comparing results derived from temperature measurements at the inner and outer plasma halves. The magnitude of the systematic error in calculated $q_0$ resulting from calibration uncertainty, as determined by comparison of similar discharges, can be up to 30%. When data from the same plasma positions are used the reproducibility is very good, typically 5%. However, since comparison with directly measured values is impossible, the absolute accuracy of the technique is unknown.

The relative merit of this program is in the comparison of discharges under differing condition - or in monitoring the time evolution of a single discharge. Provided that systematic errors in the ECE data are respected, the results are rather reliable. Additionally, the parameterisation of the temperature profile in the described fashion should be useful to other programs.
5. Description of Program

Program 'QTEST' runs in CMS with Tektronix 4025 terminal.

Program exists in module form resident in ESK as 'QTEST MODULE'

Parameter file 'DATA QDATA' must be linked.

Example: LINK USER ESK ESK <ret>
(co)py module and qdata,
QTEST <ret>
6. References

/1/ D.J. Campbell and A. Eberhagen, 6th Int. Conf. Infrared and Millimeter Waves, Miami (1981)
Contains a description of the ECE diagnostic

Derivation of equilibrium pressure profile and 'shift' of flux surfaces

Standard treatment of classical resistivity

A more precise treatment of correction to classical resistivity due to impurity ions
FIGURE 1  Example Output

Q PROFILE AT 1.00 SEC

SHOT NOS.  
9601  9606

CURRENT DENSITY (Mamp/sqM)

SAFETY FACTOR

Q=1  -8  13 cm
Q=2  -28  31 cm

FIT PARAMETERS:  
CENTRAL TEMP.  708 eV  
PROFILE CENTRE  2.2 cm  
PARABOLA POWER  1.8
Qo = 0.90  BT = 2.2 T  
Zeff = 1.19  IP = 312 KA  
BHLI/2 = 0.95  NE = 4.0 E13  
Vcalc = 0.95  LI = 1.41

Q PROFILE AT 1.25 SEC

SHOT NOS.  
9601  9606

CURRENT DENSITY (Mamp/sqM)

SAFETY FACTOR

Q=1  *NOT FOUND*
Q=2  -22  32 cm

FIT PARAMETERS:  
CENTRAL TEMP.  1799 eV  
PROFILE CENTRE  8.3 cm  
PARABOLA POWER  1.1
Qo = 1.26  BT = 2.2 T  
Zeff = 0.37  IP = 313 KA  
BHLI/2 = 2.23  NE = 4.7 E13  
Vcalc = 0.17  LI = 1.14
FIGURE 2  Go DURING CURRENT RISE

Zeit  Datum  Schuss  Diagn.  Signal  Massstab
14.39.21  15-JUL-83  9617  30  JPLAS  0.50000E+06
FIGURE 3  Q₀ DURING CURRENT FALL

![Graph showing plasma current vs. time with a sawtooth end and specifications: Zeit 08.21.41  Datum 04-AUG-83  Schuss 9877  Diagn. 30  Signal JPLAS  Massstab 0.5000E+06]
**Figure 5** Example Output

**Q Profile at 1.25 Sec**

**Shot Nos.**
9601

**Current Density (Mamp/sqM)**

<table>
<thead>
<tr>
<th>Radius (cm)</th>
<th>3.0</th>
<th>2.0</th>
<th>1.0</th>
</tr>
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<tbody>
<tr>
<td>-40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Safety Factor**

\[ Q=1 \quad \text{**NOT FOUND**} \]
\[ Q=2 \quad -18 \quad 29 \text{ cm} \]

**Fit Parameters:**
- Central Temp. 1630 eV
- Profile Centre 8.4 cm
- Parabola Power 0.8
- \( Q_0 = 1.51 \)
- \( B_T = 2.2 \text{ T} \)
- \( Z_{\text{eff}} = 0.25 \)
- \( I_P = 312 \text{ KA} \)
- \( B^2 I/2 = 2.25 \)
- \( N_E = 4.7 \times 10^13 \)
- \( V_{\text{calc}} = 0.16 \)
- \( L_I = 0.93 \)

---

**Q Profile at 1.25 Sec**

**Shot Nos.**
9606

**Current Density (Mamp/sqM)**

<table>
<thead>
<tr>
<th>Radius (cm)</th>
<th>3.0</th>
<th>2.0</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40</td>
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<tr>
<td>+40</td>
<td></td>
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</tr>
</tbody>
</table>

**Safety Factor**

\[ Q=1 \quad \text{**NOT FOUND**} \]
\[ Q=2 \quad -23 \quad 32 \text{ cm} \]

**Fit Parameters:**
- Central Temp. 1813 eV
- Profile Centre 8.2 cm
- Parabola Power 1.2
- \( Q_0 = 1.19 \)
- \( B_T = 2.2 \text{ T} \)
- \( Z_{\text{eff}} = 0.93 \)
- \( I_P = 313 \text{ KA} \)
- \( B^2 I/2 = 2.20 \)
- \( N_E = 5.1 \times 10^{13} \)
- \( V_{\text{calc}} = 0.18 \)
- \( L_I = 1.19 \)