Model Calculations of the
Tearing Instability Associated with
AC-Modulation of the Plasma-Current in Tokamaks

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Abstract

The tearing mode stability of the radial profiles of current density resulting from the skin effect accompanying AC modulation is calculated using a computer code.
1. Introduction

On the Pulsator tokamak at Garching an experiment was performed in which an AC component was superposed on the plasma current \([14,15]\). To interpret and plan these experiments, a computer program to calculate the current distributions accompanying the AC modulation taking into account the skin effect was developed. The experiments showed that AC modulation of high amplitude impairs the plasma stability and can lead to disruption. Since tearing modes are believed to be responsible for the disruptive instability, calculations on the stability of these modes were performed.

This report presents a detailed description of the computer program and its results. Section 2 deals with the skin effect; Section 3 is a discussion of the stability of tearing modes. The heating effects associated with AC modulation are discussed in Section 4. Sections 5–8 document the computer program.
2. Radial Current Profile

Figure 1a shows a schematic drawing of the essential features of the experimental configuration. The ohmic heating transformer incorporates some supplementary windings in addition to the standard windings. These windings are powered by an AC-generator which is switched on at time $t_1$ during a tokamak discharge. As shown in Fig. 1b, the plasma current, $I_p$, and the loop voltage, $V_{PL}$, consist approximately of a stationary component originating from the standard power supply and, for $t \geq t_1$, of an AC component. In the calculations it is assumed that the AC component of the plasma loop voltage is periodic. The plasma current is calculated from the plasma loop voltage, $V_{PL}$. Immediately after the AC is switched on, the plasma goes through a transient state which lasts for a few cycles; afterwards, the AC component of the plasma current is also periodic.

The penetration of the alternating current into the plasma column is calculated under the following simplifying assumptions:

1. The toroidal plasma is treated like a cylinder.

2. The displacement current is neglected because of the low frequency of the AC component.

3. Because of the strong toroidal magnetic field, motions of the plasma are neglected so that Ohm's law has the simplified form

$$J_z = \sigma E_z \quad ,$$

(2.1)

where $J_z$ and $E_z$ denote the $z$ components of the current density
and the electric field, the z direction being that of the plasma axis. \( \sigma \) is the electric conductivity.

4. Since only relatively low frequencies are applied in the experiment, the real DC component of the so-called AC conductivity \([12]\) can be used and the imaginary conductivity terms are neglected. By restricting ourselves to the DC component we neglect certain time dependences, such as the heating effect connected with the AC modulation. This is justified by the fact that such a heating effect has not been experimentally observed, probably because of insufficient modulation amplitude.

It is thus assumed that

\[
\sigma \text{ is real and time independent.} \quad (2.2a)
\]

For the radial dependence of the conductivity we use the ansatz

\[
\sigma = \sigma_o \left[ 1 - V \hat{r}^2 \right]^P, \quad (2.2b)
\]

where \( \hat{r} = r/a \) with \( a \) the limiter radius. The parameter \( V \) allows us to vary the conductivity at the plasma boundary. The conductivity on axis, \( \sigma_o \), and the profile number, \( P \), are fitted to conductivity profiles de-

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1Comparison with the transport code of Düchs and McKenney show that the assumption 2.2a is, at least theoretically, not well justified.
duced from Thomson scattering measurements. The parameters \( q_0 \), \( V \) and \( P \) have to satisfy additional constraints so that the DC-component of the plasma current, \( I_{\text{PLO}} \), equals the experimental value and that the safety factor, \( q \), on axis is of order unity. The DC component, \( E_o \), of the electric field does not depend on \( r \); consequently, the plasma direct current is

\[
I_{\text{PLO}} = 2\pi E_o \int_0^a dr \, r \, \sigma .
\]  

(2.4)

The DC component, \( E_o \), corresponds to the DC component of the plasma loop voltage

\[
V_o = 2\pi R \, E_o ,
\]  

(2.5)

where \( R \) is the major torus radius. Denoting the DC component of the current density by \( j_o(r) \) it follows that

\[
 j_o(r) = j_o(0)(1 - \frac{r^2}{R^2})^P
\]  

(2.6)

with

\[
 j_o(0) = V_o \frac{\sigma_0}{2\pi R} .
\]  

(2.7)

Substituting Eq. (2.2b) in Eq. (2.4) yields

\[
 j_o(0) = (P+1)V \cdot \frac{I_{\text{PLO}}}{\pi a^2}
\]  

(2.8)
if the condition

\[(1 - \nu)^{P+1} << 1 \quad (2.9a)\]

is satisfied. The parameters \(V\) and \(P\) consequently appear in the combination \((P+1) \cdot V\) [for example, in Eq. (2.19)]. It is therefore useful to define the quantity, \(Q\),

\[Q = (P+1)V - 1 \quad . \quad (2.9b)\]

The other constraint is that the safety factor, \(q\), on axis is approximately one. From the definition of \(q\) the current density on axis can be shown to be

\[j_0(0) = \frac{2B_{\text{tor}}}{\mu_0 q(r=0) R} \quad (2.10)\]

where \(R\) is the major radius of the plasma and \(\mu_0\) the permeability of the vacuum. Inserting this expression into Eq. (2.7), one gets

\[V_{o \sigma} = \frac{4\pi B_{\text{tor}}}{\mu_0 q(r=0)} \quad . \quad (2.11)\]

Conditions such as \(q(r=0) \approx 1\) thus fix \(j_0\) and hence the product, \(V_{o \sigma} \).

Making use of the simplifying assumptions (1 to 4), a simple parabolized differential equation for the electric field component, \(E_z\), can be derived from Maxwell's equations:

\[\frac{1}{r} \frac{3}{3r} \left[ r \frac{3}{3r} E_z \right] = \mu_0 \sigma \frac{3}{3t} E_z \quad (2.12)\]
with boundary conditions

$$E_z(r=a) = \frac{V_{PL}}{2\pi R} \quad (2.13)$$

and

$$\frac{\partial}{\partial r} E_z(r=0) = 0 \quad . \quad (2.14)$$

Condition (2.14) assures that the electric field in the plasma center at $r=0$ is continuous. The initial condition at time $t = t_1$ is

$$E_z(r) = \frac{V_0}{2\pi R} \quad , \quad (2.15)$$

where $V_0$ is the DC component of the plasma loop voltage $V_{PL}$ (see Fig. 1b). The parabolic differential equation (2.12 - 2.15) was solved by a numerical method which is described in detail in Section 5.

We now discuss some typical current profiles which were calculated for Pulsator discharges. As will be seen shortly, they can be scaled easily to machines of arbitrary size. With sinusoidal time dependence of the loop voltage, the current density, $j(r,t)$, will also become sinusoidal after a transitional time period of a few cycles; i.e.

$$j(r,t) = j_0(r) + j_1(r) \sin \left[ \omega t + \phi(r) \right] \quad , \quad (2.16)$$

where the DC component, $j_0$, originates from the standard power supply and the AC component, $j_1$, from the AC generator and where $\phi(r)$ denotes
the phase with respect to the sinusoidal part of the loop voltage.

Some typical examples for the variation of the amplitude of the current density with radius are shown in Fig. 2. We see that the AC current is induced predominantly in the outer layers of the plasma because of the skin effect. On the other hand, only very little current can flow directly at the plasma boundary because the temperature there is low and the resistivity is high. The AC-current density consequently, if ω is sufficiently high, see (2.20), has a maximum at a position, \( r_{\text{max}} \). In order to estimate the position, \( r_{\text{max}} \), of the current-density maximum for tokamaks of arbitrary size with widely differing conductivity profiles, we have derived an approximation formula for \( r_{\text{max}} \) obtained from a few dozen examples of our skin effect model

\[
\hat{r}_{\text{max}} = \frac{1}{0.175 (1 + 0.135 Q^2)^{1/2} + 2.6 Q \hat{\lambda}}.
\]  

(2.19)

In Eq. (2.19), \( r_{\text{max}} \) is normalized on the plasma radius and depends on two dimensionless parameters, namely on \( Q \), which was previously discussed in Eq. (2.9b), and on the normalized skin depth, \( \hat{\lambda} \).

\[
\hat{\lambda} = \frac{1}{\alpha \mu_0 \sigma_0 \omega}.
\]  

(2.18)

Equation (2.19) is valid in the range

\[
0.01 \leq \hat{\lambda} \leq 0.2/\sqrt{\mu_0} \quad , \quad 1 \leq Q \leq 10.
\]  

(2.20)
For $\lambda \leq 0.005$ the $r_{\text{max}}$ calculated according to Eq. (2.19) is approximately by 5% small. For $\lambda = 0$ one should get $r_{\text{max}} = 1$, but this limiting case is not correctly reproduced by Eq. (2.19). The RHS of the interval (2.20) describes the case that $j_1$ is only very slightly larger in the vicinity of $r_{\text{max}}$ than it is for small $r$. When $\lambda$ is further increased, the maximum vanishes and $j_1$ shows similar qualitative behaviour to the DC profile, $j_0$. For $V = 1$ the inaccuracy of the approximation formula is about 0.01 to 0.02. For $V = 0.7$ the $r_{\text{max}}$ calculated according to Eq. (2.19) is already approximately 0.03 too small for $\lambda = 0.025$. This effect is caused by the plasma boundary layer, described by $V$, which was neglected when formulating Eq. (2.19). As soon as $\lambda$ becomes larger than the RHS of Eq. (2.20), the AC profile, $j_1$, becomes more and more like the DC profile, $j_0$, and the subsidiary maximum at $r_{\text{max}}$ vanishes.

The penetration of the alternating current into the plasma depends strongly on the conductivity at the boundary, i.e., in our model on the parameter, $V$, from Eq. (2.2). Examples of this are shown in Fig. 4. It is found that the AC profile, $j_1$, varies more strongly with $V$ than the DC profile, $j_0$. It is unfortunate that the conductivity at the boundary is very poorly known experimentally. This causes considerable difficulty in comparing our calculated results with experimental results.

So far only the amplitude of $j_1$ of the AC current density has been discussed. Because of the phase factor, $\Phi(r)$, appearing in Eq. (2.16), the actual time development of the current density profile is slightly more complicated. In Fig. 3 we show the motion of a current density profile during one cycle of the AC modulation using the same conductivity profile as in Fig. 2. The solid line is the DC profile, $j_0(r)$, which represents the current density profile up to the time $t = t_1$. The transient process is not shown in Fig. 3; only the motion after the transition period as described by Eq. (2.16). The motion has a certain vague similarity with the motion of a rope which is tied at the top left and which is under tension and made to oscillate at the bottom right. The waves generated travel from the bottom right to the top left,
their amplitude becomes smaller, till it vanishes at the top left. The wavelength of the oscillation is of the order of the skin depth, $\lambda = 1/\sqrt{\sigma_0 \omega}$, at the position, $r_{max}$.

The results shown in Figs. 2 to 8 were calculated to match experimental data from the Pulsator experiment, where the minor radius $a$ was 0.11 cm, the central temperature was 0.6 keV, the central conductivity, $\sigma_0$, was $1.4 \times 10^7 \ \Omega^{-1} \ m^{-1}$, and the modulation frequency, $\omega$, in the range 100 Hz $< \omega < 2000$ Hz. Because of the dimensionless formulation of the equations, the figures can be easily scaled to other radii. Consider, e.g., the case $\omega = 1000$ Hz in Fig. 2. For a device with $a = 1$ m and $T_e(0) = 6$ keV, the curve would be exactly the same for $\omega = 0.4$ Hz, (scaling $\lambda = 1/\sqrt{\mu_0 \sigma_0 \omega}$). This frequency seems to be very small in comparison with discharge duration and the AC affects only the outermost layer of the plasma.
3. Investigation of Tearing Instabilities

The previous section demonstrated that the radial current density profile can be strongly varied by AC modulation (see, for example, Fig. 3). Since the stability of tearing modes (e.g., Ref. [2]) is strongly dependent on the shape of the current density profile, AC modulation can be used as a powerful tool for the comparison of theoretical predictions of tearing mode theory and experimental observations. This provided the motivation for investigating the AC modulated current density profiles for tearing instability.

The tearing instability develops preferentially at a plasma radius, \( r_s \), where the safety factor, \( q \), is rational;

\[
q(r_s) = \frac{m}{n} \quad \text{with} \quad m = 2 \text{ or } 3 \\
\quad \quad \quad \quad \quad n = 1 \text{ or } 2 .
\]

(3.1)

Here \( m \) and \( n \) are the integer mode numbers of a Fourier expansion in the poloidal and toroidal angles, \( \theta \) and \( \phi \). In typical tokamak discharges

\[
q(r=0) \lesssim 1 , \\
q(r=a) \gtrsim 2.3 \text{ to } 8 .
\]

(3.3)

In order to estimate the stability behaviour, we used a routine, written by Lackner and based on the works of Furth, Rutherford, and Selberg [2] and of White, Monticello, Rosenbluth, and Waddell [3].
This program calculates for a given current density profile, \( j_z(r) \), the instability parameters, \( \Delta' \), \( r_s \), and \( w \), the significance of which is explained as follows:

1) \( \Delta' \) is the jump of the logarithmic derivative of the eigenfunction of the mode at the resonant q-surface and determines the linear growth rate, \( \gamma \). The relations between \( \Delta' \) and \( \gamma \) was first described by Furth, Killeen, and Rosenbluth \[1\]. A nice representation is given in Batemans monograph \[4\] which gives \[Eq. (10.2.20)\]

\[
\gamma = 0.55 \, \Delta'^{0.8} \, \eta^{0.6} \, (kB^2)_{\alpha}^{0.4} \, \rho^{-0.2},
\]

where \( \eta \) is the resistivity, \( \rho \) the plasma density, and \( B \) the magnetic field strength.

\( \Delta' \) also determines the marginal instability. According to Furth, Rutherford, Selberg \[2\], for a cylindrical zeropressure plasma, the instability criterion is

\[
\Delta' > 0. \quad (3.5)
\]

Glasser, Johnson, and Greene \[5\] have also taken the pressure and toroidal effects into account. Instead of Eq. (3.5), they obtain the criterion for marginal instability

\[
\Delta' > \Delta_{\text{crit}}. \quad (3.6)
\]

In typical tokamak discharges one obtains a small improvement of the stability, i.e.,
\[ a_{\text{crit}} \approx 1 \text{ to } 3 \hspace{1cm} (3.7) \]

2) \( r_s \) is the radius of resonant q surface.

3) \( w \) is the (full) width of the saturated magnetic island associated with the tearing mode; the island width normalized to the plasma radius is defined as \( \hat{w} \), where

\[ \hat{w} = w/a \hspace{1cm} (3.7a) \]

The island width, \( w \), depends on the nonlinear saturation amplitude of the tearing mode. According to White, Monticello, et al. \[3\], island formation leads to flattening of the current profile in the vicinity of the resonant q surface. The island continues to grow until one has \( \Delta' = 0 \) for the flattened current profile. The nonlinear saturation width of the island can be estimated according to Jaenicke, Wobig, and Callen \[9,16\] in a very rough approximation by calculating the value \( w \) for which

\[ \frac{\partial}{\partial r} \left[ \psi_1 \left( r \frac{w}{2} \right) - \psi_1 \left( r - \frac{w}{2} \right) \right] = 0 \hspace{1cm} (3.8) \]

where \( \psi_1 (r) \) is the radial part of the linear eigenfunction of the mode expressed in terms of the helical flux,

\[ \psi = \psi_0 (r) + \psi_1 (r) \cos(m\theta + n\phi) \hspace{1cm} (3.9) \]

According to the present view a magnetic island grows until its width
attains the value, \( w \). If it then overlaps an adjacent island, there is ergodization of the magnetic field lines near the islands. This leads to disruption of the magnetic confinement of the plasma \([6,7]\). Overlapping of the \( m = 2, n = 1 \) mode with the \( m = 3, n = 2 \) mode appears to be particularly dangerous.

We now discuss some typical results of these calculations. First, we investigate the stability of the DC profile. For typical Pulsator current profiles the \( m = 2, n = 1 \) mode is marginally unstable while the higher \( m \) modes are stable. For example, for the profile

\[
j_o = 7 \cdot 10^6 (1 - 0.716 r^{-2})^{5.76}
\]

(3.10)

values of \( a \Delta' \) listed in Table I are obtained.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( n )</th>
<th>( a \Delta' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>4.3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-1.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-4.5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>-7.4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>-8.0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>-8.4</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>-9.1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>-11.0</td>
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<tr>
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</tr>
<tr>
<td>5</td>
<td>4</td>
<td>-16.0</td>
</tr>
</tbody>
</table>
If AC modulation is applied, the DC current density as well as $d/dr j_z$ at the resonance surface, and hence $\Delta'$, will vary periodically. According to the instability criterion, $\Delta' > \Delta'_{\text{crit}}$, the modes therefore become unstable with increasing AC amplitude in the sequence given in Table I. Figure 5 shows an example of a sinusoidal AC modulation. This figure was obtained with the ordinary differential equation (7.2) (see Sec. 7 on the GOEL program). Using the partial differential equation (2.12) to (2.15) and plotting $\Delta'$ for the 7th cycle, one obtains $\Delta'$ values approximately 20% lower than the corresponding $\Delta'$ values of Fig. 5, indicating that the relaxed state has not yet been completely attained after 7 cycles. Unlike Fig. 5, Figs. 6, 7 and 8 were calculated according to Eq. (2.12) to Eq. (2.15).

For low amplitudes the modulation of $\Delta'$ is sinusoidal (e.g., the curve $\Delta I = 5$ kA in Fig. 5). For high amplitudes, on the other hand, $\Delta'$ is not sinusoidal (e.g., $\Delta I = 20$ kA in Fig. 5). This non-sinusoidal modulation of $\Delta'$ is due in part to the strong radial motion of the resonant surface. As a consequence, the mean value $\langle \Delta' \rangle$, i.e., $\Delta'$ averaged over a cycle of the AC modulation, increases strongly with the AC voltage, indicating that the plasma is destabilized. This behaviour was found typical for all sinusoidal AC modulations.

Figure 6 shows examples in which the frequency was varied. For higher frequencies and comparable current modulation the mode 3/1 becomes unstable (see Fig. 6b). At even higher frequency, the 4/1 mode is destabilized, so that AC heating of the plasma boundary layers is likely to be difficult.
The physical interpretation of the quantity, $\Delta'$, and its mean value, $<\Delta'>$, should now be further discussed. As shown previously, $\Delta' > \Delta_{\text{crit}}$ corresponds to instability [see Eq. (3.6)], and $\Delta'$ determines the linear growth rate [see Eq. (3.4)]. It is also known [13] that the range of validity of the linear theory is restricted to very low amplitudes. For somewhat higher amplitudes the modes grow according to Rutherford's formula [13]

$$\frac{d\hat{\omega}}{dt} = \frac{\hat{\omega}}{t_i} = \frac{1.66}{\sigma \mu_o a^2} \Delta' \ (w=0) \ .$$

(3.12)

Even this reduced growth rate, $t_i$, is, in most cases, much smaller than the period, $t_{AC}$, of the AC modulation:

$$t_i \ll t_{AC} \ .$$

(3.13)

In the case of Fig. 5a, for example, one has

$$t_A = \frac{2\pi}{\omega} = 2.5 \ \text{ms},$$

$$t_i = 0.6 \sigma \mu_o a^2 w/\Delta' = 0.1 \text{ to } 0.2 \ \text{ms},$$

where $\sigma = 0.2 \times 10^7 \ (\text{ohm}^{-1} \ \text{m}^{-1})$ at the resonance surface,

$$\mu_o = 4\pi \times 10^{-7} \ (\text{V} \ \text{s} \ \text{A}^{-1} \ \text{m}^{-1}),$$

$$a\Delta' = 14 \ \text{ (for } \Delta I = 10 \ \text{kA}),$$

$$a = 0.11 \ \text{m} \ .$$
\[ \hat{\omega} = 0.1 \]

Consequently, as soon as a mode has become unstable, the saturation amplitude is very quickly attained — if condition (3.13) is satisfied —, and the magnetic islands develop almost immediately to their full width.

It is therefore of interest to discuss the behaviour of the island width \( w \). First, there is no direct connection between \( w \) and \( \Delta' \). If \( \Delta' \) is negative, the plasma is stable with \( w = 0 \), but as soon as \( \Delta' \) becomes positive a magnetic island of finite width, \( w \), forms. In our calculations, however, we have found both large \( w \) together with small \( \Delta' \) and, conversely, small \( w \) with large \( \Delta' \). It therefore seems advisable to discuss the behaviour of \( w \) from examples.

Figure 7 shows the position, \( \hat{r}_s \), of the resonant surface and the curves, \( \hat{r}_s + w/2 \) plotted versus "time", \( t \), for typical Pulsator conditions. Without AC modulation the mode \( m/n = 2/1 \) is unstable at all times and an island with normalized width, \( \hat{\omega} \approx 0.2 \), forms. The island width becomes slightly modulated by a small AC current (\( \Delta I = 3.5 \) kA, Fig. 7a). For larger AC modulation (\( \Delta I = 5.2 \) kA, Fig. 7b) \( \Delta' \) becomes negative for a short time so that the island vanishes. At other time phases the \( 3/2 \) mode also becomes unstable. With even larger AC modulation (\( \Delta I = 10 \) kA, Fig. 7c) the islands of the \( 2/1 \) and \( 3/2 \) modes overlap. This overlapping is caused, among other things, by the fact that the resonant surfaces approach each other closely during the time phase in which the two modes are unstable and simultaneously have large islands. Our model thus predicts that at certain time phases stabilization of the \( 2/1 \) mode will occur and that at other time phases the \( 2/1 \) and \( 3/2 \) modes overlap. It should be possible
to verify these predictions experimentally because the Mirnov oscillations should vanish during the stabilized phase and disruptions should occur during the overlapping phase.

In summary, it can be stated that AC modulation varies periodically the current profile, and hence the stability of the plasma periodically improves and deteriorates. The stability depends on a series of factors e.g., the motion of the resonant surfaces, the steepness of the current density profile and many others. With sinusoidal modulation these factors generally tend to cause a deterioration of the stability properties. It was hoped, however, that the most unfavourable effects will be minimized by appropriate non-sinusoidal modulation. We therefore investigated numerous non-sinusoidal modulations. We can summarize the most important results: with carefully tailored sawtooth modulation of the current (see Fig. 8a) overlapping of the magnetic islands of various modes did occur at much higher amplitudes, $\Delta I$, than for comparable sinusoidal modulation. To obtain this result, we introduced short voltage pulses at regular time intervals, $2\pi/\omega$, for the AC component of the plasma loop voltage, $V_{PL}$. Between two voltage pulses we kept $V_{PL}$ constant. These voltage spikes (Fig. 8a, curve $V_{PL}$) lead to a sawtooth-like current modulation (Fig. 8a, curve $I_{PL}$). In Fig. 8 the amplitude of the voltage pulses is chosen such that the amplitude, $\Delta I$, of the current modulation is the same as that in Figs. 6a and 7c for sinusoidal modulation. Comparison of the two cases shows that:

1) The mean value $<\Delta'>$ is about 6.2 in both cases, irrespective of the shape of the current modulation. 2) With sinusoidal AC modulation (Fig. 7c) overlapping of the magnetic islands occurs; with the sawtooth modulation of the current chosen in Fig. 8 there is no overlapping.
There is no "genuine" stabilization involved in this case since $\langle \Delta' \rangle$ is roughly as large as in the comparable sinusoidal case, but overlapping is prevented because of the larger separation of the resonant surfaces during the unstable phase compared with the sinusoidal case.

The opposite case that $\langle \Delta' \rangle$ is reduced can also be obtained by applying negative voltage pulses and an inverted sawtooth modulation of the current \[15\]. In this case, however, overlapping occurs for much smaller amplitudes than for sinusoidal modulation.

While we could improve the stability properties compared to sinusoidal modulation, we have found no case where the stability was improved over the DC case, considering both the $\langle \Delta' \rangle$ criterion and the overlapping criterion.
4. Heating Power

Finally, we want to discuss the heating effects associated with AC-modulation. The primary motivation for this investigation has been to check, whether the absence of heating effects in the Pulsator experiment is consistent with the prediction of our model. Due to the skin effect the AC will preferentially heat the outside of the plasma column. The AC heating competes with the DC-input energy which is transported through the outside region. We therefore took as a criterion that the AC heating power has to be comparable with a reasonable fraction of the DC power input in order to have a noticeable effect.

First, we study how the heating power depends on the parameters. In our model the total heating power, averaged over a cycle, is

$$
<H> = \frac{1}{2\pi} \int_0^{2\pi} d(\omega t) \int_0^a dr r^2 \sigma E_z^2.
$$

(4.1)

With sinusoidal modulation one has

$$
E_z = E_o + E_1 \cos(\omega t + \phi),
$$

(4.2)

where $E_1$ and $\phi$ depend on $r$. The DC component, $E_o$, depends neither on space nor time. We can therefore divide the heating power into a DC component, $H_o$, and an AC component, $H_1$:

$$
<H> = H_o + H_1,
$$

(4.3)

where

$$
H_1 = 2\pi R \int_0^a dr r \sigma E_1^2
$$

(4.4)
\[ H_i \approx 2 \pi R a^2 E_i^2(a) \hat{\lambda}^x \]  
\[ x = 1.213 + 0.34 \ln Q \]  

The two parameters, \( \hat{\lambda} \) and \( Q \), have already been introduced in Eqs. (2.9b) and (2.18); they are the "essential" parameters of the problem. The approximation formula (4.6) was obtained from a several examples, the inaccuracy being approximately 10%. Equations (4.2 to 4.7) are only valid in the relaxed state. It should be mentioned that this state is reached often only after many cycles and that the heating power, \( H_i \), depends very sensitively on how exactly the relaxed state is attained. In the example in Fig. 7b, \( H_i \) in the 7th period is about 9 kW, but in the 37th period is about 5 kW. The latter value corresponds very well to the relaxed state.

The amplitude of the AC modulation and consequently also the AC-heating power is limited by the development of instabilities, which can lead to disruptions. In our model this is equivalent to the overlapping of two islands of different helicities. We therefore have investigated at what AC amplitudes the islands come into contact with each other and what heating power corresponds to these AC amplitudes. In the example shown in Fig. 7b, the DC-power input is 120 kW, while the AC power is 5 kW. Moreover, Fig. 7b shows that the 3/2 and 2/1 islands are so close that the AC-power input is at its upper limit. This and similar calculations showed that for sinusoidal AC modulation the maximum ratio \( H_i/H_0 \) is only a few percent. This result is in good agreement with the experiment.

We mention that for higher frequencies a larger value for the ration
$H_1/H_0$ can be obtained without overlap. For instance, for the same parameters as in Fig. 7b, but with $\omega/2\pi = 10^4$ Hz, we obtain $H_1 \approx H_0$. In this case the heating power input is limited because the 3/1 and the 4/1 mode overlap. At the high frequencies, however, it is only the extreme outer plasma boundary layer that is heated by the AC-modification and the energy confinement in this layer is very short.

Finally, we obtained considerably larger values of $H_1$ with sawtooth modulation. This is due to the prevention of overlapping as discussed in Sec. 3. In the example shown in Fig. 8, overlap does not happen until $H_1/H_0 \geq 20\%$, i.e., more than twice the value of a comparable sinusoidal modulation (Fig. 7b). But even in this more favorable case, the heating effects probably would be only marginally observable in our experiment.

Before we proceed with a detailed description of the computer programs, we want to summarize the main conclusion of our computation regarding the physics of the AC-modulation.

(a) Through AC modulation the current density profile in tokamak plasmas can be quite significantly altered, which in turn significantly modifies the stability of tearing modes. One of the surprising features of the experiments [14,15] has been the fact that very large AC amplitudes could be applied without disrupting the plasma. The computations showed that although the instability parameter, $\Delta'$, may become very large, $(\Delta' \approx 10-15)$, overlap of islands does not necessarily occur. The AC amplitudes which are needed to produce overlap are - according to our computations - comparable to the experimentally observed amplitudes.
(b) Overlap should occur when the saturation width, \( w \), of the island becomes very large and when the distance between resonant surfaces becomes small. Both factors seem to be of equal importance. By suitable shaping of the applied AC-voltage, e.g., by applying a sawtooth shaped current modulation rather than sinusoidal one, we were able to move resonance surface apart during the unstable period of the AC modulation.

This method can reduce some of the adverse effects of sinusoidal AC-modulation. However, we were not able to obtain better stability than with the DC-current profile.

(c) The calculations predict that disruptions caused by changes in the current density profile during AC occur before significant heating is produced by the AC-modulation. The best heating efficiency was obtained by sawtooth modulation, and the AC power input was about 20\% of the DC power input.
FIG. 1a

Scheme of experimental arrangement

FIG. 1b

Plasma current $I_{PL}$ and loop voltage $V_{PL}$ versus time $t$
FIG. 2

Current Densities, $j_0$ (= DC) and $j_1$ (= AC) normalized on 1, versus $\tilde{r} = r/a$, for

$$\sigma = 1.4 \times 10^7 \left[ 1 - 0.716 \tilde{r}^2 \right]^{5.76} \text{[Ohm m]$^{-1}$}$$

$$\frac{\omega}{2\pi} = 100 ; 300 ; 1000 \text{ sec}^{-1}$$
Current Density Distribution during AC-Modulation

\[ I = 55 \text{ kA} \]
\[ \Delta I = 20 \text{ kA} \]
\[ f = 400 \text{ Hz} \]
\[ \sigma_o(r) = \sigma_o \left[ 1 - 1.716 \left( \frac{r}{a_L} \right)^2 \right] 5.76 \]
\[ q_o(0) = 0.91 \]
\[ q_o(a_L) = 4.4 \]

\[ \omega t = 1.5 \pi \]
\[ \omega t = \pi \]
\[ \omega t = 0.5 \pi \]
\[ \omega t = 0 \]

\[ r/S(2/1) \]
Current densities $j_0$ (= DC) and $j_1$ (= AC) versus $\tilde{r} = r/a$, for

$$\sigma_1 = 1.4 \times 10^7 \left[ 1. - 1. \tilde{r}^2 \right]^{3.85} \text{[Ohm m]}^{-1}$$

$$\sigma_2 = 1.4 \times 10^7 \left[ 1. - 0.716 \tilde{r}^2 \right]^{5.76} \text{[Ohm m]}^{-1} \quad (\tilde{r} \geq 0.7)$$

$$\sigma_3 = 1.4 \times 10^7 \left[ 1. - 0.2 \tilde{r}^2 \right]^{23.2} \text{[Ohm m]}^{-1} \quad \text{(dashed)}$$

$V_0 = 2.2$ Volt

$V_C = 50.$ Volt

$$\frac{\omega}{2\pi} = 300 \text{ sec}^{-1}$$

$I_{PL} = 55$ kA

Note: $V_{PL} = V_0 + V_C \cos \omega t$. 
FIG. 5

\( a \Delta' \) versus \( \omega t \)

\( \frac{\omega}{2\pi} = f = 400 \text{ sec}^{-1} \)

for \( I_{PL} = 55.\text{kA} \)

\( j_0(r) = 7 \times 10^6 \left( 1.40.716 \frac{r^2}{2} \right)^{5.76} \) \( \text{[A m}^{-2} \]\n
FIG. 6

\( a \Delta' \) versus \( \omega t \)

for \( I_{PL0} = 55.\text{kA} \) and \( j_0(r) \) from FIG. 5

calculated from eq. (2.12 for the 7th cycle)

FIG. 6a

\( \frac{\omega}{2\pi} = 400.\text{sec}^{-1} \) (vgl. FIG. 5)

\( \Delta I = 10.\text{kA} \)

FIG. 6b

\( \frac{\omega}{2\pi} = 4000.\text{sec}^{-1} \)

\( \Delta I = 8.\text{kA} \)
FIG. 7

\( \hat{r}_s, \hat{r}_s + \frac{\hat{w}}{2} \text{ und } \hat{r}_s - \frac{\hat{w}}{2} \) versus \( \omega t \)
for three values of the current modulation \( \Delta I \).

\( \hat{r}_s \) = radius of the resonant q surface,
\( \hat{w} \) = full width of the magnetic island,
\( \hat{r}_s \) and \( \hat{w} \) are normalized to the plasma radius.
The other parameters are the same as in FIG. 6a.

FIG. 7a \( \Delta I = 3.5 \, kA \)

FIG. 7b \( \Delta I = 5.2 \, kA \)

FIG. 7c \( \Delta I = 10.0 \, kA \)
Sawtooth modulation of the plasma current.
All parameters are the same as those in FIGS. 6a and 7c.

Fig. 8a
plasma current $I_{\text{PL}}$ and
loop voltage $V_{\text{PL}}$
versus $\omega t$.
The amplitude of the
current modulation is
$\Delta I = 10$. kA.

Fig. 8b
$\hat{r}_s$, $\hat{r}_s = \frac{\dot{\omega}}{2}$ und $\hat{r}_s - \frac{\dot{\omega}}{2}$
versus $\omega t$. 
APPENDIX: Computer programs

5. GEOPAR program

The figures shown in sec. 3 are calculated with the GEOPAR program.
This program is now described in detail to allow the reader to re-
produce these figures or to produce similar figures with other para-

ters. The program has the following objectives:

1) input and preparation of necessary parameters;

2) Calculation of
   radial mesh points,
   conductivity,
   initial conditions for the field component $E_z$,
   shape factor COSOMZ determining modulation,
   current density profile,
   heating power,
   instability parameters (from FURTH subroutine),
   $E_z$ at equidistant time intervals (from PARCYL subroutine)

3) drawing of figures in the ZEICH subroutine; for this purpose the
   quantities of interest are stored in the two-dimensional array $Z$
   at the end of the SIGFRI subroutine.

1) Meaning of input parameters

We start with the integer parameters:
M(J) and N(J) are the mode parameters, i.e. the wave numbers of a Fourier
   expansion of the helical flux in the poloidal angle and
   toroidal angle; see table below in the section of SIGFRI;

NPR(J) the times at which current profiles and instability para-
   meters should be calculated, in units of the time step
   size DT defined below.
Such a parameter array is necessary because, on the one hand, DT has to be fairly small for numerical calculation of \( E_z \), and, on the other, current profiles and instability parameters are only needed after relatively large time intervals;

\[
\begin{align*}
LMA &= \text{number of radial mesh points;} \\
NP &= \text{number of AC cycles;} \\
NT &= \text{number of time intervals per cycle.}
\end{align*}
\]

The real parameters are now given in alphabetic order:

\[
\begin{align*}
A &= \text{radius for which the profile function for the conductivity vanishes (in metres); (see Fig. 9);} \\
AL &= \text{limiter radius \( a \) (in metres)} \\
&\quad \text{the connection with parameter} V \text{ from eq.(2.2) is explained below;} \\
BTOR &= \text{toroidal field in tesla (1 tesla = } 10^4 \text{ gauss);} \\
FR &= \text{AC frequency in } s^{-1} = \frac{\omega}{2\pi} \\
P &= \text{profile number from eq. (2.2)} \\
RG &= \text{major radius in metres;} \\
SIGO &= \sigma_0 \text{ from eq. (2.2);} \\
V_C &= \text{AC voltage in volts;} \\
V_G &= \text{DC voltage in volts;} \\
R_{\text{SOND}} &= \text{distance of sond from the plasma axis, this parameter is not used in this paper.}
\end{align*}
\]

**Other parameters**

\[
\begin{align*}
DR &= \text{space step} = AL / (LMA -1); \\
DT &= \text{time step} = 1 / (FR \ NT) = 2\pi / (\omega \ NT)[s]; \\
MO &= \mu_0 \text{ from eq. (2.10)} = 4\pi \cdot 10^{-7} \text{ in units } \frac{Vs}{Am};
\end{align*}
\]
QF = time independent component of the safety factor q.

For q the following equations are valid:

\[ q = \frac{r B_{tor}}{R B_{pol}} = \frac{2\pi B_{tor}}{\mu_0 R} \frac{r^2}{I(r)}, \tag{5.1} \]

where \( Q_F = \frac{2\pi B_{tor}}{\mu_0 R} \) is the space and time independent component of q.

and

\[ I(r) = 2\pi \int_0^r dr' r' j_z(r') \tag{5.2} \]

\( I(r=a) \) is thus equal to the plasma current \( I_{PL} \).

VERS = \( \left[ \frac{A_L}{A} \right]^2 = V \) from eq. (2.2).

FIG. 9 illustrates the significance of AL and A and V.
2) \text{COSOMZ}

The plasma loop voltage is periodic for \( t \geq t_1 \). We write in the form

\[
V_{pl} = V_G + V_C \cos \text{COSOMZ}.
\]

This equation is also valid for \( E_z(a) \) because of eq. (2.13). With sinusoidal AC modulation one has according to eq. (2.16):

\[
\text{COSOMZ} = \cos \omega t;
\]

hence the name COSOMZ. The AC voltage does not, however, have to be sinusoidal; it may also be sawtooth-like or otherwise. The FORM subroutine calculates COSOMZ for one cycle, i.e. for \( J = 1 \) to \( J = NT \). Meanwhile there are many such FORM subroutines which are stored in various AMOS segments, e.g. sinusoidal modulation in FORM6; sawtooth modulation in FORM2; etc.

3) \text{SIGFRI}

The SIGFRI subroutine calculates for a given field \( E_z(r) \):

- the current density profile \( j_z(r) \)
- the heating power \( \int dr \, r \, E_z(r) \, j_z(r) \, 4\pi^2 R/1000 \) [kW]

and, by means of CALL FURTHER, the instability parameters

\[
D_L(K) = \Delta'a,
\]

\[
RQ(K) = \text{radius of the resonant Q surface, divided by a}
\]

and \( \text{DE}(K) = \text{half island width} = \frac{\hat{d}}{2} \).
Here the index K gives the number of pairs of mode parameters m, n which have to be taken into account. In the curves shown in this report these are:

<table>
<thead>
<tr>
<th>K</th>
<th>m</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

SIGFRI is called at two points:

1) as soon as the initial values for $E_z$ are available;

2) in the DO-7 loop after CALL PARCYL for the values of $E_z$ calculated by PARCYL; but, of course, only for those times NPR(LS) for which the current density and instability parameters are to be calculated; hence the inquiry in statement ISN 0046 in the main program.

We require various radial mesh point distributions because:

In the FURTH subroutine
the index $L = 1$ has to be assigned the value $\hat{r} = 0$ and
the index $L = LMA$ has to be assigned the value $\hat{r} = 1$;
the plasma radius thus has to be normalized to 1.

In the PARCYL subroutine
the value $r = 0$ has to be in the centre between $L = 1$ and $L = 2$
the value $r = a$ has to be in the centre between $L = LMA$ and $LMA-1$;
$R(L = 1)$ is thus negative.

The parameter JWRITE regulates the printout of the results:
with JWRITE = 0 nothing is printed out,
with JWRITE $\geq 2$ ZEIT, current profile, q etc. are printed out.
After calculation of the current profile CALL FURTH is given for calculating the tearing instability parameters; the quantities of interest are stored in array Z.

\[ \mathcal{E}_Z \]

\[ \mathcal{G}_Z \]

\[ \mathcal{G}_L \]

\[ \mathcal{L}_L \]

\[ \mathcal{M}_L \]

\[ \mathcal{K}_L \]

\[ \mathcal{E}_L' \]

\[ \mathcal{L}' \]

\[ \mathcal{R}_L \]

\[ \mathcal{R}_M \]

\[ \mathcal{R}_1 \]

\[ \mathcal{R}_2 \]

\[ \mathcal{R}_{L+1} \]

\[ \mathcal{R}_{L+1} \]

Figure 10 shows the positions of the R mesh points which are needed for PARCYL, with \( R_1 \) negative. The hatched surface illustrates the calculation of the current, which with equidistant \( R_L \) is as follows:

\[ I(R_M) = 2\pi \int_0^{R_M} r \sigma(r) \mathcal{E}_Z(r) \, dr \]

\[ \approx \int_{R_L}^{R_L} \mathcal{E}_Z = 2\pi \int_{R_L}^{R_M} r \sigma(r) \mathcal{L}_L \cdot \mathcal{L}'_L \cdot \mathcal{L}_L' \cdot \mathcal{E}_Z \cdot \mathcal{E}_Z' \cdot \mathcal{L}_L'. \]

6. PARCYL subroutine

The name stands for PARabolic partial differential equation with CYLindrical symmetry. The PARCYL subroutine calculates \( E_z \) at time \( t + DT \) from \( E_z \) at time \( t \).
We now describe the numerical method by which this is done.

Given is the parabolic partial differential equation

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial F}{\partial r} \right] = S \frac{\partial F}{\partial t}
\]

with the boundary conditions \( \frac{\partial F}{\partial r} = 0 \) for \( r = 0 \),
\( F = G \) for \( r = a \).

\( G = G(t) \) and \( S = S(r,t) \) are given functions.

Required \( F = F(r,t) \).

In order to solve the equation numerically we approximate the given differential equation by a difference equation. For this purpose we introduce equidistant time mesh points and space mesh points \( r_j \); the \( r_j \) need not be equidistant. The point \( r_j = 0 \) has to be left out because of the factor \( \frac{1}{r} \) in the given differential equation. Let

\[ F_j = F(r_j, t) \]

and

\[ \hat{F}_j = F(r_j, t + DT) \].

The \( F_j \) are known, either as initial values or from the previous time step; the \( \hat{F}_j \) are to be calculated from the \( F_j \). The difference equation can be divided into recursion formulae, which are solved successively from the boundary conditions.

First we present the discretization scheme and then the formulation and rearrangement of the difference equation.

**Discretization scheme**

![Diagram of the radial mesh points](image)

**FIG. 11** Distribution of the radial mesh points.
Figure 11 shows the distribution of the radial mesh points. It must hold that
\[ r_1 = -r_2 \]
and
\[ 2a = r_{N+1} + r_N. \]
The other mesh points can be arbitrarily chosen. To approximate
\( \partial F / \partial r \), we require the auxiliary quantities
\[ r_{J+1/2} = \frac{1}{2} (r_J + r_{J+1}), \]
\[ d_J = r_J - r_{J-1}, \]
and
\[ d_{J+1} = r_{J+1} - r_J. \]

Formulation of difference equation

We approximate
\[ \frac{\partial F}{\partial t} \text{ by } \frac{1}{DT} \cdot \left[ F_J - F_{J-1} \right], \]
\[ S \text{ by } \frac{1}{2} \cdot \left[ S_J + S_{J-1} \right], \]

\[ \frac{3}{\partial r} \left[ r \frac{\partial F}{\partial r} \right] \text{ by } \frac{1}{d_{J+1/2}} \left( \left[ r \frac{\partial F}{\partial r} \right]_{r+1/2} - \left[ r \frac{\partial F}{\partial r} \right]_{r+1/2} \right), \]
\[ \frac{3}{\partial r} \left[ r \frac{\partial F}{\partial r} \right] \text{ by } \frac{r_{J+1/2}}{2d_{J+1/2}} \left[ F_{J+1} - F_J + F_{J+1} - F_J \right] \]
and
\[ - \left[ r \frac{\partial F}{\partial r} \right] \text{ by } \frac{r_{J-1/2}}{2d_J} \left[ F_{J-1} - F_J + F_{J-1} - F_J \right]. \]

There is thus a similarity to, for example, the implicit scheme given by RICHTMYER /11/. Inserting these approximations in the partial differential equation yields
\[ A_J \hat{F}_{J+1} + B_J \hat{F}_J + C_J \hat{F}_{J-1} = D_J \]

where
\[ A_J = r_{J+1/2} \left[ d_{J+1} (S_J + S_J^{-1}) r_J d_{J+1/2} \right] \]
\[ C_J = r_{J-1/2} \left[ d_{J} (S_J + S_J^{-1}) r_J d_{J+1/2} \right] \]
\[ B_J = -A_J - C_J - R_T \]
\[ R_T = 1 / DT \]

and
\[ D_J = A_J (F_J - F_{J+1}) + C_J (F_J - F_{J-1}) - R_T F_J \]

These equations are further rearranged to give the recursion formulae
\[ \hat{F}_J = H_J \hat{F}_{J+1} + U_J \]

with
\[ U_J = (D_J - U_{J-1} C_J) / (B_J + C_J H_{J-1}) \]

and
\[ H_J = -A_J / (B_J + C_J H_{J-1}) \]

From the boundary condition \( F_1 = F_2 \)
it follows that \( H_1 = 1 \)
and \( U_1 = 0 \).

This allows all \( U_J \) and \( H_J \) to be calculated in succession. From
the boundary condition \( F(r = a) = G \)
it follows that
\[ \frac{1}{2} (\hat{F}_N + \hat{F}_{N+1}) = \hat{G} \]

and
\[ \hat{F}_{N+1} = \frac{2 \hat{G} - \hat{U}_N}{1 + H_N} \]

This also allows the \( F_J \) to be calculated in succession. This
algorithm was provided by D. DUECHS.
REAL*4  IO, JZ(202), LAMBDA, MO
DIMENSION COSOMZ(637), S(57), NPR(77)
COMMON /SIF/ EZ(57), R(57), RD(202), SIG57, Z117, 77)
1, A, AL, BTDOR, DR, DT, FR, OM, P, QF, PG, RSND, SIGO, VERS, VC, VG, ZEIT
2, MI(4), N1(4), LMA, NP, NT
COMMON /FUR/ DE(5), DL(5), RQ(5), SHEAR(5)
READ 180, (HI(J), J=1,3), (NI(J), J=1,3)
PRINT 91
READ 180, (NPRI(J), J=1,27)
PRINT 180, (NPRI(J), J=1,27)
READ 312, LMA, NP, NT, A, AL, BTDOR, FR, P, R, RSND, SIGO, VC, VG
L1 = LMA + 1
MO = 1.25664E-6
SIGO = SIGO * E7
OM = 6.2832 * FR
UMFANG = 6.2832 * RG
QF = 6.2832 * BTDOR / (MO * RG)
DR = AL / (LMA - 1.)
DT = 6.2832 / (OM * NT)
VERS = (AL/A)**2
EC = VC / UMFANG
EG = VG / UMFANG
RT = 1. / DT
DO 2 L=1, L1
R(I) = DR*(L - 1.5)
RD(I) = R(I) / AL
BASIS = 1. - VERS* RD(L)**2
SIGI = 0.
IF (BASIS < 1.5E-6) GO TO 1
SIGI = SIGO * BASIS**P
1 M = MO * SIGI
2 EZ(I) = EG
CALL FORM (COSOMZ, NT)
PRINT 92
PRINT 110, (COSOMZ(J), J=1, NT)
ZEIT = 0.
EL = 0.
LS = 1
CALL SIGFRI (JZ, Q, LS)
DO 7 JP=1, NP
DO 7 JT=1, NT
ZEIT = ZEIT + DT
ED = EG + EC* COSOMZ(JT)
CALL PACSYL (EZ, S, DR, EL, ED, RT, LMA)
J = JT + (JP-1)*NT
IF (NPR(LS) < NE = J) GO TO 7
LS = LS + 1
CALL SIGFRI (JZ, Q, LS)
IZ = 1. / (AL SORT * SIGO * FR = 6.2832)
PRINT 98
PRINT 95
PRINT 313, LMA, NP, NT, A,AL, BTOR, FR, P, RG, RSOND, SIGO, VC, VG
PRINT 101, VERS, DT, LAMBDA
PRINT 93
PRINT 94, (M(J), N(J), J=1,3), (M(J), N(J), J=1,3)
1, (M(J), N(J), J=1,3), (M(J), N(J), J=1,3)
DO 8 L=1, LS
8 PRINT 107, (Z(K,L), K=1,16)
   DM = 0.
   HM = 0.
   SM = 0.
   DO 9 L=1, LS
   SM = SM + (Z(14,L) + Z(14,L-1)) * (Z(1,L) - Z(1,L-1))
   DO 9 L=1, LS
   HM = HM + (Z(4,L) + Z(4,L-1)) * (Z(1,L) - Z(1,L-1))
   DO 9 L=1, LS
   SM = SM * 0.0755775
   DM = DM * 0.0755775
   DM = DM * 0.0755775
   SM = SM * 0.0755775
   PRINT 103, DM, SM, HM
   CALL ZEICH(Z, 7, LS)
   GO TO 10
   91 FORMAT(10H, NPR)
   92 FORMAT(10H, CDOSMZ)
   93 FORMAT(123H, D E L S, I N S E L B R E I T E)
   1 RADIUS DER Q-FLAECHEN
   2 B / B (E) (KA) (KW) (GAU)
   3 SSS)
   94 FORMAT(9H, OMT,15,11,15,11,15,11,19,11,16,11,16,11,19,11,15,1
   1,15,11,19,11,15,11,27H)
   95 FORMAT(102H, LMAX, NP, NT, A, AL, BTOR, FR
   1 P, RG, RSOND, SIGO, VC, VG)
   98 FORMAT(1H1)
   101 FORMAT(12H, VERS=,F7.2,12H)
   103 FORMAT(46H, VALUES AVERAGED OVER ONE PERIOD)
   1 DM=,F4.1,
   147X, 2F8.1)
   1, 4F8.2)
   110 FORMAT(10F12.4)
   111 FORMAT(3F12.3, 1P11E10.2)
   180 FORMAT(19I4)
   312 FORMAT(3I4, 10F6.2)
   313 FORMAT(1I0, 2I6, 7F8.2, 1PE9.2, 0P2F8.2)
   END
SUBROUTINE SIGFRI ( JZ, Q, LS)
REAL*4 IR, JZ(202), JZH
DIMENSION R,F(202)
COMMON /FUR/ DE(5), DL(5), RQ(5), SHEAR(5)
COMMON /SIF/ EZ(57), R(57), RD(202), SIG(57), Z(17,77)
1 ,A,AL, BTOR,DR,DT, FR,OM,P,OF, RG,RSOND, SIGO, VERS, VC,VG, ZEIT
2 , MI(4), NI(4), LMA, NP, NT

ISN 0007
ISN 0008
ISN 0009
ISN 011
ISN 012
ISN 013
ISN 014
ISN 015
ISN 016
ISN 017
ISN 018
ISN 019
ISN 020
ISN 021
ISN 022
ISN 023
ISN 024
ISN 025
ISN 026
ISN 027
ISN 028
ISN 029
ISN 030
ISN 031
ISN 032
ISN 033
ISN 034
ISN 035
ISN 036
ISN 037
ISN 038
ISN 039
ISN 040
ISN 041
ISN 042
ISN 043
ISN 044
ISN 045
ISN 046
ISN 047
ISN 048
ISN 049
ISN 050
ISN 051
ISN 052
ISN 053
ISN 054
JWRITE = 0
OFT = OM * ZEIT
IF (JWRITE .LE. 1) GO TO 1
PRINT 98
PRINT 101, LS, ZEIT, OMT
PRINT 92
PRINT 313, LMA, NP, NT, A, AL, BTOR, FR, P, RG, RSOND, SIGO, VC, VG
PRINT 94
PRINT 111, VERS
PRINT 93

1 S = 0.
HEIZ = 0.
DO 5 L = 1, LMA
    RM = 0.5* (RL + R(L+1))
    RF(L) = 0.5* (RD(L) + RD(L+1))
    SIGM = 0.5* (SIG(L) + SIG(L+1))
    EZM = 0.5* (EZ(L) + EZ(L+1))
    JZ(L) = SIGM * EZM
    SER = 0.
    IF (L .LE. 1) GO TO 4
    SER = SIG(L) * EZ(L) * R(L)
        IF (L .LE. 1) GO TO 4
        SER = SJ + SER
    HEIZ = HEIZ * SER * EZ(L)
    IR = 6.2832 * DR * SJ
    Q = OF * RM**2 / ABS( IR + 1.E-7)
    IF (JWRITE .LE. 1) GO TO 5
    PRINT 111, RF(L), RM, EZ(L), JZ(L), IR, Q

5 CONTINUE
    HEIZ = HEIZ * DR * 0.009478 * RG
    CALL FURTHER (RF, JZ, LMA, Q, -1., 100., 0., MI, NI, 3)
    RSP = AL / RSOND
    FAKTOR = 1.25E+6 * RSOND * BTOR / (AL * RG * IR)
    DO 7 K = 2, 4
        K1 = K + 3
        K2 = K + 6
        K3 = K + 9
        M = M(K-1)
        Z(K1, LS) = DL(K-1)
        Z(K2, LS) = 9Q4K-1)
        Z(K3, LS) = SHEAR(K-1) * RQ(K-1) * DL(K-1)**2 * AL**2 * FAKTOR
                    * M * RSP**M * 100.
    7 Z(K4, LS) = DT(K-1)
    Z(14, LS) = OMT
    Z(15, LS) = HEIZ
    Z(16, LS) = 2.E-3 * IR / RSOND
ISN 0055
ISN 0056
ISN 0057
ISN 0058
ISN 0059
ISN 0060
ISN 0061
ISN 0062
ISN 0063
ISN 0064

Z(17, LS) = 3.14
RETURN

92 FORMAT(/102H LMAX NP NT A AL BTOR FR
1 P RG RSOND SIGO VC VG)
1 Q)

93 FORMAT(/60H RD RM EZ JZ IR

94 FORMAT(/10H VERS)

ISN 0064

ISN 0065

ISN 0066

ISN 0067

ISN 0068

ISN 0069

ISN 0070

ISN 0071

ISN 0072

ISN 0073

ISN 0074

ISN 0075

ISN 0076

ISN 0077

ISN 0078

ISN 0079

ISN 0080

ISN 0081

ISN 0082

ISN 0083

ISN 0084

ISN 0085

ISN 0086

ISN 0087

ISN 0088

ISN 0089

ISN 0090

ISN 0091

ISN 0092

ISN 0093

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ISN 0096

ISN 0097

ISN 0098

ISN 0099

ISN 0100

ISN 0101

ISN 0102

ISN 0103

ISN 0104

ISN 0105

ISN 0106

ISN 0107

ISN 0108

ISN 0109

ISN 0110

ISN 0111

ISN 0112

ISN 0113

ISN 0114

ISN 0115

ISN 0116

ISN 0117

ISN 0118

ISN 0119

ISN 0120

ISN 0121

ISN 0122

ISN 0123

ISN 0124

ISN 0125

ISN 0126

ISN 0127

ISN 0128

END

ISK 0002
SUBROUTINE PARCYL ( F, S, DR, G, GD, RT, N)

DIMENSION F(57), FD(57), S(57), SD(57)
1 F(N+1) = 2.0*G - F(N)
1 J=1,N
1 DRS = DR**2 * (J - 1.5) * (S(J) + S(J))
1 A(J) = (J - 1.0) / DRS
1 C(J) = (J - 2.0) / DRS
1 B(J) = -A(J) - C(J) - RT
1 FM = F(1)
1 IF ( J, LE, 2) GO TO 1
1 FM = F(J-1)
1 D(J) = A(J)*((F(J) -F(J+1)) + C(J)*(F(J) -FM) - F(J)*RT
1 H(1) = 1.
1 U(1) = 0.
1 J=2,N
1 BCH = B(J) + C(J) *H(J-1)
1 U(J) = (D(J) - C(J)*U(J-1)) / BCH
1 H(J) = -A(J) / BCH
1 FD(N+1) = (2.0*GD - U(N)) / (1.0 + H(N))
1 J=1,N
1 K = N + 1 - J
1 FD(K) = H(K) *FD(K+1) + U(K)
1 J=1,N
15 F(J) = FD(J)
1 RETURN
1 END
SUBROUTINE FURTHER
C TEARING MODE STABILITY IN STRAIGHT CYLINDER
C FOLLOWING GLASSER, FURTHER, RUTHERFORD
C STOPPED IN KAL:FURTHER,FURTHER; FILE SHARED BY ALL AMOS USERS
C
C MEANING OF INPUT PARAMETERS
C
C RADIAL COORDINATE NORMALIZED BY PLASMA RADIUS A
C RI **** RADIAL COORDINATE OF POINTS IN WHICH PLASMA
C CURRENT DENSITY IS GIVEN
C AJ **** VALUES OF CURRENT DENSITY IN POINTS RI (IN ARBI-
C TRARY UNITS)
C NPIN **** NUMBER OF POINTS OF RI, AJ
C **************** R(I) MUST BE 0, R(NPIN) MUST BE 1.**********
C QB **** Q OF PLASMA CURRENTS AT PLASMA BOUNDARY
C BW **** RADIUS OF CONDUCTING WALL (SPECIFY A NEGAT. VAL-
C UE IF NO CONDUCTING WALL PRESENT)
C NP **** NR OF GRID POINTS USED FOR COMPUTING MARGINAL
C MHD MODE IN BOTH INTERVAL (0,R SINGULAR) AND IN
C (R SINGULAR, 1]
C JOTA0 **** JOTA OF EXTERNAL WINDINGS
C HI, HII **** VECTORS OF THE (M,N) VALUES FOR THE MODES TO BE
C EXAMINED
C NMN **** NUMBER OF (M,N) PAIRS TO BE EXAMINED
C
REAL*4 RI(202), AJ(202), AJZ(202), DU1(202), DU2(202), Q1(202), Q2(202),
X A1(808), R(202), Y(202), YSING(202), YT0T(202),
X RL(202), RP(202), YDL(202), YDR(202)
REAL*4 JOTA0
DIMENSION XP(21), YP(21), XZ(2), YZ(2), XX(2), YX(2), TEXT(5)
COMMON /FUR/ DE(5), BR(5), RQ(5), SHEAR(5)
INTEGER*4 M(4), N(4), IOUTP(4)
DATA ERROR/.1, E-05/

RHS(RD) = -(AKS - G*(RD-RS))/(RD-RS)
X + ALL *(AKS*(AKS+2MP1/RD-G*(RD-RS)))
X + (RD-RS)*ALL *(AKS*AKS*(2MP1/RD-G*(RD-RS)))
X + 1.5*AKS*AKS + 2MP1/RD*AKS
X + .5*(RD-RS)*2MP1/RD*AKS

C SPLINING OF CURRENT PROFILE
C
DO 1 K=1,5
DE(K) = 0.
BRI(K) = 0.
RQ(K) = 0.
SHEAR(K) = 0.
AJ2(1) = 0.
D2 = PI(NPIN) - PI(NPIN-2)
D1 = RI(NPIN) - RI(NPIN-1)
AJ2(NPIN) = (AJ(NPIN-2)*D1**2 - AJ(NPIN-1)*D2**2)
X + AJ(NPIN) = (D2**2-D1**2)
X
/(D2*2*D1-D1**2*D2)
call cubic3(npin,ri,aj,aj2,du1,du2)

C

computation of Q-profile and rescaling of j

Q(1) = 0.
D0 111 I = 2, NPIN
D0 112 I = 1, NPIN
A = (AJ2(I)-AJ2(I-1))/(6.*DX)
B = .5*AJ2(I-1)
C = (A(I-1)-A(I-1))/DX - DX/6.* (AJ2(I) + 2.*AJ2(I-1))
D = A(I-1)
Q(I) = Q(I-1)
X + (((0.2A*DX + 0.25*(A*RI(I-1)+B))*DX
X + (8*RI(I-1)+C)/3.)*DX
X + 0.5 *(C*RI(I-1)+D))*DX
X + D*RI(I-1))*DX

111 continue
coef = 1./(6.2831853*Q(NPIN))
D0 112 I = 2, NPIN
A(I) = AJ2(I)*COEF
A(I) = AJ2(I)*COEF
Q(I) = Q(I)/1.*Q(I)*JOTAQ
Q(I) = Q(I)/1.*Q(I)*JOTAQ

113 continue
Q2(I) = 0.
Q2(NPIN) = 2.*QB
QBB = QB/(1.+ QB*JOTAQ)
call cubic3(npin,ri,q,q2,du1,du2)

C

choice of H, N - mode and location of resonant surface

IMN = 0
D0 201 IMN = IMN + 1

116 IF(IMN.GT.NMN)return
M = H(I+1)
N = H(I)
D1L=0.
G = (B+LT+0.) GO TO 221
GAM = M*(1.+B**(2)*M))/(1.-B**(2)*M))
GO TO 222
GAM = -M
GO TO 222
221 continue
A2MP1 = 2*M+1
QRES = FLOAT(M) / FLOAT(N)
IF (M.LT.N*Q(1)) GO TO 201
IF (M.GT. N*QBB) GO TO 500
RRES = (QRES-Q(I1))/(Q(NPIN) - Q(I1))

202 call c0d1nt(R1,R2,NPIN,RRES,RQ,RDR,IFLAG)
C SOLUTION FOR THE REGION R < RS

602 RS = RES
605 DR = RS/(NP+.5)
608 R(I) = .5*DR
611 DO 211 I = 2,NP
614 211 R(I) = R(I-1) + DR.

621 CALL CDINT (R1, AJ, AJ2, NPIN, RS, AR, ADR, IFLAG)
624 CALL CDINT (R1, Q2, NPIN, RS, QR, QDR, IFLAG)
627 SHEA = QDR / QR**2
630 AKS = -6*2831853 * QP*ADR/(QP*RS*N*QDR) * M
633 AKSP = AKS - A2MP1/RS
636 R(NP+1) = RS

C SET-UP OF COEFFICIENT MATRIX ACCORDING TO 18 OCT/4 - 1976

652 DL = 2*R(I1)
655 CALL CDINT (R1, AJ, AJ2, NPIN, R(I1), AR, ADR, IFLAG)
658 CALL CDINT (R1, Q, Q2, NPIN, R(I1), QR, QDR, IFLAG)
661 G = 6*2831853 * QP*ADR/(R(I1)*QP*(H-N*QR)) * M
664 ALL = ALOG (RS-R(I1))
667 AI(2) = -1. -A2MP1 - G*DL**2
670 AI(3) = 1. + A2MP1
673 AI(4) = DL*2*RHS(R(I1)) - DL*(1-A2MP1)*AKS*(ALOG(RS) + 1. X -AKSP * RS * (ALOG(RS) + .5))

683 DO 301 I = 2,NP
686 NQ = 4*(I-1)
689 DR = R(I+1)-R(I)
692 DL = R(I) - R(I-1)
695 CALL CDINT (R1, AJ, AJ2, NPIN, R(I), AR, ADR, IFLAG)
698 CALL CDINT (R1, Q, Q2, NPIN, R(I), QR, QDR, IFLAG)
701 G = 6*2831853 * QP*ADR/(R(I)*QP*(H-N*QR)) * M
704 ALL = ALOG(RS-R(I))
707 AI(0+1) = (2. - A2MP1/R(I)*DR)*DR
710 AI(0+2) = (-2. + A2MP1/R(I)*DR-DL) - G*DR*DL*(DR+DL)
713 AI(0+3) = (2. + A2MP1/R(I)*DL)*DL
716 AI(0+4) = DL*DR*(DR+DL)*RHS(R(I))
719 DO 301 CONTINUE

725 CALL LGLS(A1, Y, NP-1, 0,.6)
728 DO 302 I = 1,NP
731 ALL = ALOG(RS-R(I))
734 YSING(I) = 1. +AKS*(R(I)-RS)*ALL
737 X + .5*AKSP*AKSP = (R(I)-RS)**2*ALL
740 YTOT(I) = (R(I)/RS)**M * (Y(I) + YSING(I))

745 DO 302 CONTINUE
752 YS1 = (Y(NP-1) - 4.*Y(NP))/(2.*(R(NP) - R(NP-1)))

755 NPP1 = NP+1
758 DELL = DR
761 DO 321 I = 1,NP
764 RL(I) = R(I)

C
321  Y(I) = YTOT(I)
        R(LNP+1) = RS
        Y(NP+1) = 1.
        CALL DDT3(DR,Y,YDL,NPP1,IER)
        C STATEMENTS TO ADD IF DPSI/DR / PSI USED
        C DO 325 I = 1,NPP1
        C 325  YDL(I) = YDL(I)/Y(I)

        YDL = C0.
        YDD = 1.*

        C SOLUTION FOR REGION R > RS
        C
        NR = NP + 1
        DR = (1.-RS)/(NR-.5)
        NRP1 = NR + 1
        R(I) = RS
        DO 401 I = 2,NRP1
        R(I) = R(I-1) + DR
        DO 402 I = 2,NR
        NO = 4*(I-2)
        DR = (R(I+1)-R(I))
        DL = R(I) - R(I-1)
        CALL DQINT(RI,C,Q2,NPIN,R,I,QR,QDR,IFLAG)
        G = 6.2831853 * QR*ADR/ (R(I)*QB*(M-N*QR))*M
        ALL = ALOG(R(I)-RS)
        A1(NO+1) = (2.* - A2MPI/R(I))*DR)*DR
        A1(NO+2) = (-2.* + A2MPI/R(I)*DR-DL) - G*DR*DL/(DR+DL)
        A1(NO+3) = (2.* + A2MPI/R(I)*DL)*DL
        A1(NO+4) = DR*DL*(DR+DL)*RHS(R(I))
        401 CONTINUE
        A1(I) = 0.*
        NO = 4*(NR-1)
        A1(NO+1) = (-1.* + 5*DR*(GAM-M))
        A1(NO+2) = 1.* - 5*DR*(GAM-M)
        A1(NO+3) = 0.*
        A1(NO+4) = -DR*(AKS*(ALL+1.)*AKS*AKSP*(1.*-RS)*(ALL+.5)
        X = (GAM-M)*[1.*AKS*(1.-RS)*ALL + 5*AKS*AKSP*(1.-RS)**2*ALL]
        CALL LGLLS(A1,Y,NR-1,0.,6)
        402 CONTINUE
        A1(I) = 0.*
        J = I - 1
        ALL = ALOG(R(I)-RS)
        YSING(J) = 1.*AKS*(R(I)-RS)*ALL
        X = 5.*AKS*AKSP*(R(I)-RS)**2*ALL
        YTOK(I) = (R(I)/RS)**M *(Y(J) + YSING(J))
        404 CONTINUE
        411 YS2 = (4.*Y(I) - Y(2))/(2.*R(I-R(1))
        N9 = NR - 1
        N9 = NR - 1
        SHEAR(IHM) = YTOK(N9) * SHEA
        421 YSING(I) = YTOK(I-1)
CALL DET3(CR,YS1,YR,APP1,IER)  
C STATEMENTS TO ADD IF DPSI/DR / PSI USED  
C DO 425 I = 1,NAPP1  
C 425 YDR(I) = YDR(I)/YSING(I)  
C------------------------------------------------------------------------------------------
C------------------------------------------------------------------------------------------
C------------------------------------------------------------------------------------------
C Eğer DELS < 0 ise GÖT TO 210  
C DEL = DELL  
C IF (DELR.LT.DEEL) DEL = DELR  
C DLL = DEL  
C REL = RS-DLL  
C RER = RS+DLL  
C CALL ALI(REL,RL,YDL,YDEL,NPP1,IER)  
C CALL ALI(RER,RR,YDR,YDER,NPP1,IER)  
C DELSL = YDER - YDEL  
C IF (DELSL.GT.0) GOTO 461  
C GÖT TO 210  
C 461 CONTINUE  
C DLL = DLL + DEL  
C REL = RS-DLL  
C RER = RS+DLL  
C CALL ALI(REL,RL,YDL,YDEL,NPP1,IER)  
C CALL ALI(RER,RR,YDR,YDER,NPP1,IER)  
C DELSL = YDER - YDEL  
C X = M*AKS/RS  
C X*([(RS+DLL)/RS]**(1-M))*((DLL+ALOG(DLL)+0.5*AKS*DLL)**2*ALOG(DLL))  
C X = M*AKS/RS  
C X*([(RS-DLL)/RS]**(1-M))*((DLL+ALOG(DLL)-0.5*AKS*DLL)**2*ALOG(DLL))  
C X = M*AKS/RS  
C X*([(RS+DLL)/RS]**M)*((ALOG(DLL)+1-0.5*AKS*2*D*L+ALOG(DLL)+DLL))  
C X*([(RS-DLL)/RS]**M)*((ALOG(DLL)+1-0.5*AKS*2*D*L-ALOG(DLL)+DLL))  
C IF (ABS(DESL).GE.(100.*ERROR*DELS)) GOTO 472  
C 462 CONTINUE  
C GÖT TO 210  
C 472 IF (DESL) 473, 462, 474  
C 473 DLL = DLL - DEL  
C DEL = 0.5*DEL  
C GÖT TO 461  
C 474 IF ((REL.GT禄RL(R2))*AND*(RER.LT.RR(NP-1))) GOTO 461  
C GÖT TO 210  
C------------------------------------------------------------------------------------------
C------------------------------------------------------------------------------------------
C------------------------------------------------------------------------------------------
C SCLUTION IF Q RES IN VACUUM  
C------------------------------------------------------------------------------------------
C------------------------------------------------------------------------------------------
C------------------------------------------------------------------------------------------
C 500 NR = NP  
C IF (JOTA0=QRES,GE.1) GÖT TO 505  
C RS = SQRT(QRES/5B/11-JOTA0=QRES)  
C IF ((RS.GT.0) = AND*(RS.GT.BW)) GÖT TO 201  
C GAM = 2.*M*RS**(2*M)/(1.-RS**2**(2*M))  
C DR = 1./NR  
C R(I) = S*DR  
C NRPL = NP+1  
C GO 501 I = 2, NRPL  
C 501 R(I) = F(1-1)*DR  
C CALL ODINT (R1,AJ,AJ2,NPIN,R(I),AR,ADR,IFLAG)  
C CALL OINT (R1,QR,NPIN,R(I),QR,ADR,IFLAG)  
C G = 6.*2831853 * QR*ADR*M/ (R1)*QB*QN+QR*CR))
SUBROUTINE LGGLSS(A,N,EPS,NOUT)

C LOESUNG EINES LINEAREN GLS AX = D ; A:TRIDIAGONAL MATRIX
C AUF A WERDEN DIE KOEFFIZIENTEN JEDER GLEICHUNG
C A(I)*X(I-1)+B(I)*X(I)+C(I)*X(I+1)=D(I)
C HINTEREINANDER GE SPEICHERT.
C DIE SPEICHERBELEGUNG SIEHT ALSO SO AUS:
C 1. ZEILE VON A : A(1),A(2),..,A(N)
C 2. ** ** A: B(0),B(1),..,B(N)
C 3. ** ** A: C(0),C(1),C(2),..,C(N-1)
C 4. ** ** A: D(0),D(1),D(2),..,D(N)
C A WIRD ZERSTOERT.
C X ENTHAELT DIE BERECHNETE LOESUNG.
C EPS: 3-SCHRANKE.
C NOUT: ALSGABE KANAL.
C BEMERKUNG:
C DAS DURCHGEFUEHRTE VERFAHREN IST IN RICHMyER-MORTON:
C "DIFFERENT METHODS FOR INITIAL VALUE PROBLEMS"
C SEITE 199 FF BESCHRIEBEN.
C DIE OBIGEN BEZEICHNUNGEN ENTSPRECHEN IN RICHMyER-MORTON:
C A(J) = -C(J)/(A(1))
DL = 2*R(1)
A(2) = -1,-A2MP1 - G*DL**2
A(3) = 1+2MP1
A(4) = 0.
DD = 502 I = 2, NP
ND = 4*(I-1)
DR = R(I-1) - R(I)
DL = R(I) - R(I-1)
CALL CDINT (R,I,AJ2,NPIN,R(I),AR,ADR,IFLAG)
CALL CDINT (R,I,Q2,NPIN,R(I),QR,ODR,IFLAG)
G = 6*2831853 = QR*ADR(R(I)/QR)/(2*+M-N*QR)
A(INO+1) = (2* - A2MP1/R(I)*DR)/DL
A(INO+2) = (2* + A2MP1/R(I)*DR)/DL
A(INO+3) = (2* + A2MP1/R(I)*DL)/DL
502 A(INO+4) = 0.
NO = 4* NP
A(INO+1) = -(1*+5*DR*GAM)
A(INO+2) = 1* - 5*DR*GAM
A(INO+3) = 0.
A(INO+4) = -DR*GAM/RS**M
CALL LGGLSS(A,Y,H,P,0.,6)
DO 504 I = 1, NR
YSING(I) = 0.
YSING(I) = R(I)**M*Y(I)
ISN 238 504 CONTINUE
ISN 239 511 DD = (RS/BW)**(2*M)
IF (BKLT+0.) DD = 0.
DT = RS**(2*M)
512 DELS = M/RS *((DD+1)/(DD-1) - (Y(NR) + Y(NR+1))**RS**M - 2*DT)
X / (1**-DT) + 1.)
505 CONTINUE
210 CONTINUE
ISN 244 DE(IMN) = DELS
ISN 245 BRI(MN) = DLL
ISN 246 RQ(IMN) = RS
ISN 249 81 FORMAT(40H)
ISN 250 82 FORMAT(110,5F10.3)
ISN 251 GO TO 201
END
SUBROUTINE ODISIT(X,F,FDX,DP,XP,FP,DFP,IFLAG)
DIMENSION X(I),F(I),FDX(I)

C -----------------------------------------------
C GIVEN A FUNCTION F AND ITS SECOND DERIVATIVE FDX IN
C NP POINTS X, COMPUTES THE FUNCTION AND ITS FIRST
C DERIVATIVE IN XP
C -----------------------------------------------

IFLAG = 2
IF (XPTLX(1)) OR (XPXTX(NP)) RETURN
IFLAG = 1
DO 101 I = 2, NP
IF (XPXTX(I)) GO TO 101

101
DX = X(I) - X(I-1)

A = (FDX(I) - FDX(I-1)) / (6.*DX)
B = 0.5*FDX(I-1)

C = (F(I)-F(I-1))/DX - DX/6.*(FDX(I) + 2.*FDX(I-1))

D = F(I-1)

XX = XP -X(I-1)

FP = ((A*XX+B)*XX+C)*XX + D

FDP = (3.*A*XX + 2.*B)*XX + C

RETURN

101 CONTINUE
RETURN
END

SUBROUTINE CUBIC3(N,X,Y2,F,G)
C KUBISCHER SPLINE MIT Y* AM RANDNVORGEGBEN
C H. SPAETH, PG. 46

DIMENSION X(I),Y(I),Y2(I),F(I),G(I)

N1 = N-1
J1 = 1
H1 = C.
R1 = Y2(I1)

G(1) = 0.
F(1) = 0.

DO 3 K=1,N

IF (KLE N1) GO TO 1

H2 = 0.
R2 = Y2(N)

GO TO 2

1
J2 = K+1

H2 = X(J2) - X(K)
R2 = (Y(J2) - Y(K))/H2

Z = 1./(2.*(H1+H2) - H1*G(J1))

G(K) = Z*H2

F(K) = Z*(6.*(F2-R1)-H1*F(J1))

J1 = K

H1 = H2

R1 = R2

GO TO 3

2

CONTINUE

Y2(I1) = F(I1)

DO 4 J1 = I1,N1

K = N - J1

Y2(K) = F(K) - G(K)*Y2(K+1)

CONTINUE

RETURN
END
SUBROUTINE DET3(H,Y,Z,NDIM,IER)
DIMENSION Y(1),Z(1)
TEST OF DIMENSION
IF(NDIM-3) EQ 1,I,1
TEST OF STEPSIZE
1 IF(H) EQ 2,5,2
PREPARE DIFFERENTIATION LOOP
2 HH=5/H YY=Y(NDIM-2)
B=Y(2)+Y(2)
B=HH*(B+Y(3)-Y(1)-Y(1)-Y(1))
START DIFFERENTIATION LOOP
DO 3 I=3,NDIM
A=B
B=HH*(Y(I)-Y(I-2))
3 Z(I-2)=A
END OF DIFFERENTIATION LOOP
NORMAL EXIT
IER=0
A=Y(NDIM-1)+Y(NDIM-1)
Z(NDIM)=HH*(Y(NDIM)+Y(NDIM)+Y(NDIM)-A-A+YY)
Z(NDIM-1)=B
RETURN
ERROR EXIT IN CASE NDIM IS LESS THAN 3
4 IER=1
RETURN
ERROR EXIT IN CASE OF ZERO STEPSIZE
5 IER=1
RETURN
END
SUBROUTINE ALL (X,XARR,YARR,Y,NP,IER)
DIMENSION XARR(1),YARR(1)
IER = 1
IF ((X.LT.XARR(1)).OR. (X.GT.XARR(NP))) GC TO 201
DO 101 I = 2,NP
101 CONTINUE
Y = YARR(I-1) + (YARR(I) - YARR(I-1)) * (X - XARR(I-1)) / (XARR(I)-XARR(I-1))
GO TO 111
111 CONTINUE
RETURN
IER = -1
Y = -1,F65
RETURN
END
SUBROUTINE ZEICH ( Z, L1, L2 )
DIMENSION ( Z(77), Y1(77), Y2(77), Y3(77), Y4(77), Z(17,77) )
XMI = Z(1, L1)
YMI = -10.
YMA = +80.
X(1) = XMI
Y(1) = YMI
X(2) = XMI
Y(2) = 20.
X(3) = Z(1, L2)
Y(3) = 20.
X(4) = Z(1, L2)
Y(4) = YMI
X(5) = XMI
Y(5) = YMI
CALL FRAME ( XMI, YMI, XMA, YMA )
CALL FLOTL ( X, Y, 5 )
X(1) = XMI
Y(1) = YMI
X(2) = XMI
Y(2) = 0.
X(3) = Z(1, L2)
Y(3) = 0.
CALL PLOTLS ( X, Y, 2 )
JMA = L2 - L1 + 1
DO 1 J = 1, JMA
L = J - 1 + L1
X(J) = Z(1, L)
Y1(J) = Z(2, L)
Y2(J) = Z(3, L)
Y3(J) = Z(4, L)
Y4(J) = Z(5, L)
1 Y4(J) = Z(5, L)
CALL FLOTLS ( X, Y1, JMA )
CALL PLOTLS ( X, Y2, JMA )
CALL PLOTLS ( X, Y3, JMA )
YMI = 0.
YMA = 3.
X(1) = XMI
Y(1) = YMI
X(2) = XMI
Y(2) = 1.
X(3) = Z(1, L2)
Y(3) = 1.
X(4) = Z(1, L2)
Y(4) = YMI
X(5) = XMI
Y(5) = YMI
CALL FRAME ( XMI, YMI, XMA, YMA )
CALL FLOTL ( X, Y, 5 )
DO 3 K = 8, 1
DO 2 J = 1, JMA
L = J - 1 + L1
X(J) = Z(1, L)
Y1(J) = Z(K, L)
Y2(J) = Z(K, L) - Z(K-3, L)
Y3(J) = Z(K, L) + Z(K-3, L)
2 Y3(J) = Z(K, L) + Z(K-3, L)
CALL PLOTLS ( X, Y1, JMA )
CALL PLOTLS ( X, Y2, JMA )
CALL PLOTLS ( X, Y3, JMA )
CONTINUE
RETURN
END
7. GOEL program

This program calculates the field and current density profiles for the relaxed state with sinusoidal AC modulation. This program was used to calculate Figs. 2, 3 and 4, in which AC and DC components are compared. In the GOEL program it is assumed that the current density and field components consist of a time independent "DC component" and an "AC component" with sinusoidal time dependence (see eqs. (2.16) and (7.1)). The parabolic partial differential equation (2.12) then yields a system of two coupled ordinary differential equations (7.2) with boundary conditions for \( r = 0 \) (eq. (7.3)) and \( r = a \) (eq. (7.4)). The variables have the same meaning as in the GOEPAR program. Instead of the plasma conductivity profile number \( P \) the plasma direct current IGA is read in and the profile number \( P \) is then calculated from it. We now describe the method of numerical solution employed in the GOELER and SUCCES subroutines.

Insertion of the equation analogous to eq. (2.16)

\[
E_z(r,t) = E_o + E_\infty(r) \cos \omega t + E_S(r) \sin \omega t
\]

(7.1)

in the parabolic partial differential equation (2.12) yields the system

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{d}{dr} E_c \right] = b E_s
\]

(7.2a)

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{d}{dr} E_s \right] = -bE_c
\]

(7.2b)

with boundary conditions
\[
\frac{d}{dr} E_C |_{r=0} = 0 \quad (7.3a)
\]
\[
\frac{d}{dr} E_S |_{r=0} = 0 \quad (7.3b)
\]
\[
E_C |_{r=a} = E_a = V_{PL}/2\pi R \quad (7.4a)
\]
\[E_S |_{r=a} = 0, \quad (7.4b)\]
\[b = \mu_0 \sigma \omega \quad (7.5)\]

Here \(b\) depends via \(\sigma\) on \(r\).

The equations are solved numerically, by reducing the boundary value problem ((7.2) to (7.4)) to an initial value problem in which boundary condition (7.4) is replaced by

\[
E_c |_{r=0} = C_0 \quad , \quad (7.6a)
\]
\[
E_s |_{r=0} = S_0 \quad . \quad (7.6b)
\]

The designation "initial value problem" for eq. (7.2, 7.3 and 7.6) comes from the analogy with the initial value problem of mechanics, where the location and velocity of a mass point at time \(t = 0\) are given.

We thus have to look for the "initial values" \(C_0\) and \(S_0\).

Because the differential equation (7.2) to be solved is linear and homogeneous, the boundary values on the right are linear and homogeneous functions of \(C_0\) and \(S_0\):

\[
E_C |_{r=a} = A_{11} C_0 + A_{12} S_0 \quad (7.7a)
\]
\[
E_S |_{r=a} = A_{21} C_0 + A_{22} S_0 \quad (7.7b).
\]
For \( C_0 = 1 \) one has
\[
E_{c/r=a} = A_{11}
\]  
(7.8a)
\[
S_0 = 0 \quad E_{s/r=a} = A_{21}
\]  
(7.8b)

for \( C_0 = 0 \) one has
\[
E_{c/r=a} = A_{12}
\]  
(7.9a)
\[
S_0 = 1 \quad E_{s/r=a} = A_{22}
\]  
(7.9b)

The SUCCES subroutine solves the initial value problem. First we use SUCCES to calculate \( A_{11} \) and \( A_{21} \) according to eq. (7.8), then to calculate \( A_{12} \) and \( A_{22} \) according to eq. (7.9). We then put the boundary conditions (7.4) in the left-hand side of eq. (7.7) and solve eq. (7.7) for \( C_0 \) and \( S_0 \). The third call of SUCCES then yields \( E_c \) and \( E_s \).

8. SUCCES subroutine

The SUCCES subroutine solves the initial value problem, i.e. eqs. (7.2, 7.3 and 7.6) from Sec. 7. The arguments have the following meanings:

\[
C_A = E_{c/r=0} = \text{initial value for } E_c
\]
\[
S_A = E_{s/r=0} = \text{initial value for } E_s
\]
\[
E_{c/L} = E_{c/r=a} = \text{right-hand boundary value of } E_c
\]
\[
E_{s/L} = E_{s/r=a} = \text{right-hand boundary value of } E_s
\]

\( E_{c/L} \) and \( E_{s/L} \) are to be calculated by SUCCES routine.

The \( r \) mesh points are located so that \( R(1) = 0 \).
For small r we use the ansatz

\[ E_C = C_A + \hat{C}_2 r^2 + \hat{C}_4 r^4 \quad (8.1a) \]

\[ E_S = S_A + \hat{S}_2 r^2 + \hat{S}_4 r^4 \quad (8.1b) \]

and determine the coefficients \( \hat{C}_2 \), \( \hat{C}_4 \), \( \hat{S}_2 \) and \( \hat{S}_4 \) from the differential equation (7.2).

For \( r \geq R(3) \) we use a difference scheme:

Let \( C(K) = E_C(R_K) \);

one then has

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{d}{dr} E_C \right]_{r=R(K)} = \frac{r_{K+1/2} - r_K}{r_{K+1} - r_K} \frac{C_{K+1} - C_K}{r_{K+1} - r_K} - \frac{r_{K-1/2}}{r_K - r_{K-1}} \frac{C_K - C_{K-1}}{r_K - r_{K-1}}
\]

\[
= b \ E_S(R(K)) = b \ S_K \quad (3.2)
\]

where

\[
S_K = E_S(r=R(K)) ,
\]

\[
r_K = R(K)
\]

and

\[
r_{K+1/2} = \frac{1}{2}(R(K) + R(K+1)) .
\]

We solve the difference equation (8.2) for \( C_{K+1} \), and also the analogous difference equation describing \( E_S \) for \( S_{K+1} \). The values of \( E_C \) and \( E_S \) are known for \( K \geq K-1 \), either from the previous step or from eq. (8.1) for small r.

List of input parameters

We now review the parameters used for calculating the curves in Figs. 2 to 8. This is necessary if these curves are to be reproduced with the programs presented in this report. The respective figure captions
are not sufficient. It is very difficult, for example, with the current modulation ∆I given in Fig. 3 to arrive at the AC voltage $V_C$ used as a parameter in producing Fig. 3.

1) Constant parameters

Limiter radius $AL = 0.11$ metre
Toroidal field $BTOR = 2.8$ tesla
Plasma direct current $IGA = 55$ kA
Major radius $RG = 0.7$ metre
Conductivity (r=0) $SIGO = 1.44 \times 10^7$ ohm$^{-1}$ m$^{-1}$
DC voltage $VG = 2.2$ Volt

2) Parameters which vary from curve to curve

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Remarks:

In program GOEPAR we have used P as an input parameter instead of $IGA = I_{PLO}$ (see eq. (2.8)). From $IGA = 55$ and $A = .13$ it follows that $P = 5.76$. 
C WECHSELSTROMPROFILE, ALTERNATING CURRENT, SEGMENT  G C E L

REAL*4 IG0, J0, J1, JG, JZ, LAMBDA, MO, N1, NG
COMMON /GOE/ R(51), BA, P, V, LMAX
COMMON /SUC/ EIC(51), ES(51)

10 PRINT 98

READ 112, A, AL, BTCR, CR, FR, IG0, CMT, RG, SIG0, VC, VG
PRINT 113, A, AL, BTCR, CR, FR, IG0, CMT, RG, SIG0, VC, VG

LMAX = 1.2 + L/CR

L1 = LMAX - 1
MO = 1.25664E-6
IGA = IG0 * 1.E3
SIG0 = SIG0 * 1.E7
OM = 6.2832 * FR

CO50MT = C50 (CMT)
SINMT = SIN (CMT)

BA = MO * SIG0 * OM * AL**2
LAMBDA = SQRT (1. / BA)

V = (AL/AL)**2
EO = VC / (6.2832 * RG)
EG = VG / (6.2832 * RG)

DO 1 L=1, LMAX

1 R(L) = DR*(L-1)
RQ = R(L)**2

B(L) = BA*(1. - V*RG)**P

CALL SUCCES (1, 0,0, A11, A21)

CALL SUCCES (0, 1, 0, A12, A22)

DET = A11 * A22 - A21 * A12

GO = EO * A22 / DET
SO = - EO / A21 / DET

CALL SUCCES (CC, SO, A3, A4)
PRINT 91

PRINT 113, LAMBDA, P, V
PRINT 93

JO = SIG0 * EG

DO 3 L=1, L1

RQ = R(L)**2

SIG = SIG0 * (1. - V*RQ)**P

J1 = SIG * SQRT(EIC(51)**2 + ECL(L)**2)

JG = SIG * EC

JZ = SIG * (EG + ES(L)) * SINCMT + ECL(L) * C050MT

NG = JG / JO

N1 = J1 / JO

NG = SIG0 / EG

GO TO 10

CALL SUCCES (CC, SO, A3, A4)
PRINT 110

GO TO 10

91 FORMAT(3/IH)
LAMBDA, P, V

92 FORMAT(1/I0H)
A, AL, BTCR, CR, FR

1 IGA

93 FORMAT(1/I0H)
R, SIG, J1, N1, JG

1 NG

98 FORMAT(1/LH)

SUBROUTINE SUCCES (CA, SA, ECL, ESL)

COMMON /GOE/ R(51), BA, P, V, LMAX

COMMON /SUC/ EIC(51), ES(51)

EC(1) = CA

ES(1) = SA

LM = LMAX - 1

C2 = 0.25 + BA* SA

S2 = - 0.25 + BA* CA

C4 = - 0.0625 * BA*(S2 + P*SA)

S4 = - 0.0625 * BA*(C2 + P*CA)

R2 = R(2)**2

EC(Z) = CA + R2*(C2 + C4*R2)

ES(Z) = SA + R2*(S2 + S4*R2)

DO 3 K=2,LM

QP = (R(K+1) + R(K)) / (R(K+1) - R(K))

QM = (R(K+1) - R(K)) / (R(K+1) - R(K))

QMQP = QM / QP

T = R(K) * (R(K+1) - R(K-1)) * B(K) / QP

ECL = ECL(K) - ECL(K-1) - QMQP * T*ES(K)

ESL = ES(K) + ECL(K-1) - ECL(K) - QMQP * T*ES(K)

EC(K+1) = ECL

ES(K+1) = ESL

END

SUBROUTINE SUCCES (CA, SA, ECL, ESL)

COMMON /GOE/ R(51), BA, P, V, LMAX

COMMON /SUC/ EIC(51), ES(51)

EC(1) = CA

ES(1) = SA

LM = LMAX - 1

C2 = 0.25 + BA* SA

S2 = - 0.25 + BA* CA

C4 = - 0.0625 * BA*(S2 + P*SA)

S4 = - 0.0625 * BA*(C2 + P*CA)

R2 = R(2)**2

EC(Z) = CA + R2*(C2 + C4*R2)

ES(Z) = SA + R2*(S2 + S4*R2)

DO 3 K=2,LM

QP = (R(K+1) + R(K)) / (R(K+1) - R(K))

QM = (R(K+1) - R(K)) / (R(K+1) - R(K))

QMQP = QM / QP

T = R(K) * (R(K+1) - R(K-1)) * B(K) / QP

ECL = ECL(K) - ECL(K-1) - QMQP * T*ES(K)

ESL = ES(K) + ECL(K-1) - ECL(K) - QMQP * T*ES(K)

EC(K+1) = ECL

ES(K+1) = ESL

END

SUBROUTINE SUCCES (CA, SA, ECL, ESL)

COMMON /GOE/ R(51), BA, P, V, LMAX

COMMON /SUC/ EIC(51), ES(51)

EC(1) = CA

ES(1) = SA

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EC(Z) = CA + R2*(C2 + C4*R2)

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QM = (R(K+1) - R(K)) / (R(K+1) - R(K))

QMQP = QM / QP

T = R(K) * (R(K+1) - R(K-1)) * B(K) / QP

ECL = ECL(K) - ECL(K-1) - QMQP * T*ES(K)

ESL = ES(K) + ECL(K-1) - ECL(K) - QMQP * T*ES(K)

EC(K+1) = ECL

ES(K+1) = ESL

END

SUBROUTINE SUCCES (CA, SA, ECL, ESL)

COMMON /GOE/ R(51), BA, P, V, LMAX

COMMON /SUC/ EIC(51), ES(51)

EC(1) = CA

ES(1) = SA

LM = LMAX - 1

C2 = 0.25 + BA* SA

S2 = - 0.25 + BA* CA

C4 = - 0.0625 * BA*(S2 + P*SA)

S4 = - 0.0625 * BA*(C2 + P*CA)

R2 = R(2)**2

EC(Z) = CA + R2*(C2 + C4*R2)

ES(Z) = SA + R2*(S2 + S4*R2)

DO 3 K=2,LM

QP = (R(K+1) + R(K)) / (R(K+1) - R(K))

QM = (R(K+1) - R(K)) / (R(K+1) - R(K))

QMQP = QM / QP

T = R(K) * (R(K+1) - R(K-1)) * B(K) / QP

ECL = ECL(K) - ECL(K-1) - QMQP * T*ES(K)

ESL = ES(K) + ECL(K-1) - ECL(K) - QMQP * T*ES(K)

EC(K+1) = ECL

ES(K+1) = ESL

END

SUBROUTINE SUCCES (CA, SA, ECL, ESL)

COMMON /GOE/ R(51), BA, P, V, LMAX

COMMON /SUC/ EIC(51), ES(51)

EC(1) = CA

ES(1) = SA

LM = LMAX - 1

C2 = 0.25 + BA* SA

S2 = - 0.25 + BA* CA

C4 = - 0.0625 * BA*(S2 + P*SA)

S4 = - 0.0625 * BA*(C2 + P*CA)

R2 = R(2)**2

EC(Z) = CA + R2*(C2 + C4*R2)

ES(Z) = SA + R2*(S2 + S4*R2)

DO 3 K=2,LM

QP = (R(K+1) + R(K)) / (R(K+1) - R(K))

QM = (R(K+1) - R(K)) / (R(K+1) - R(K))

QMQP = QM / QP

T = R(K) * (R(K+1) - R(K-1)) * B(K) / QP

ECL = ECL(K) - ECL(K-1) - QMQP * T*ES(K)

ESL = ES(K) + ECL(K-1) - ECL(K) - QMQP * T*ES(K)

EC(K+1) = ECL

ES(K+1) = ESL

END

3 CONTINUE

RETURN

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R

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**FIG. 2**
References

/13/ P.H. RUTHERFORD, Phys. Fluids 16, 1903 (1973)
/14/ S. v. GOELER et al., Proceedings of Symposium on Disruptive Instabilities, Garching (1979)