Current Driven Drift Instability
in the W VII-A Stellarator

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Abstract

The instability region and growth rates of current driven drift modes in the W VII-A stellarator are calculated.

Several theoretical results (ref. 4-7) are evaluated for specific temperature and density profiles. It is found that in the outer region of the plasma column ($r > 6$ cm) collisional drift waves with wavelengths $k^\perp = 0.13 - 0.3$ cm exist. In this region also the electron thermal conductivity determined experimentally appears to be large.
Introduction

Recent experimental data concerning anomalous transport phenomena in Tokamaks and Stellarators showed\(^{1-4}\) that the anomalous heat transport could be related to current driven drift waves. The measurements in W VII-A concerning the confinement time indicate also a dependence on the drift parameter \(\xi \sim \frac{I}{\sqrt{nT_e}}\). This dependence can be related to the increase of the thermal conductivity with the current intensity, which could be explained with the existence of the current driven drift waves. In this paper, we used the most important results of the existing theory\(^{4-7}\) to estimate the possibilities of appearance of current driven drift waves in the plasma with a density and temperature profile which is typical for the W VII-A stellarator. The temperature and density profiles used for numerical calculations are represented in Fig. 1. Although the majority of the theoretical results consider the slab model of an inhomogeneous plasma, they give information on the probability of the appearance of the instabilities.

1) Influence of the Ion Viscosity

The calculations made by Ellis and Motley\(^{4}\) for the case that only the density gradient and the driving current are considered yield a first estimate of the unstable region for a collisional plasma. It was shown that a critical current exists which can be expressed for the parameter \(\xi = \frac{k_\perp}{v_e}\) in the following form:

\[
\xi_{cr} \approx \xi_1 - \xi_2
\]

where

\[
\xi_1 = A_1 \frac{k_\perp^4 k_y}{k_y}, \quad \xi_2 = A_2 \frac{k_\perp^2 k_y}{k_y}
\]

\[
A_1 = \frac{3}{10} \left( \frac{m_e}{2m_i} \right)^{\frac{1}{2}} \frac{\tau_n}{\Omega} \frac{V_e}{\Omega} \frac{\rho_i^2}{\rho_y} \quad \text{and} \quad A_2 = \frac{3}{\tau_n} \frac{\Omega \rho_y}{\rho_y}
\]
Here we used the following notations: $j$ is the longitudinal current density in the torus, $n = \text{electron (or ion) density}$, $e = \text{electric charge of the proton}$, $u = \text{longitudinal mean velocity of the electrons}$, $m_e = \text{mass of the electrons}$, $m_i = \text{mass of the ions}$, $\Omega_i = \text{ion cyclotron frequency}$, $\rho_i = \text{ion Larmor radius}$, $r_n^{-1} = \frac{1}{n} \frac{d n}{d x}$ and the components of the wave vector are $k_x$, $k_y$, and $k_\parallel$. We used also $k_\perp = \sqrt{k_x^2 + k_y^2}$. The x-axis is oriented in the direction of $-\text{grad } n$.

The instability appears if

$$\bar{\xi} > \bar{\xi}_{cr}$$  \hspace{2cm} (2)

A critical current exists if

$$\frac{\bar{\xi}_1}{\bar{\xi}_2} = C \frac{k_\perp^2 k_\parallel^2}{k_y^2} > 1$$

with

$$C = \frac{A_1}{A_2} = \frac{1}{10} \left( \frac{m_e}{2 m_i} \right)^{1/2} \frac{r_n^2 v_e^2}{\rho_i^2 \Omega_i^2}$$

The numerical values of $C$ for the temperature and density profiles represented in Fig. 1 are given in Table 1.

**Table 1**

<table>
<thead>
<tr>
<th>$r$ (cm)</th>
<th>0</th>
<th>1.2</th>
<th>2.4</th>
<th>3.6</th>
<th>4.8</th>
<th>6</th>
<th>7.2</th>
<th>8.4</th>
<th>9.6</th>
<th>10.8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (cm$^{-2}$)</td>
<td>$9 \cdot 10^6$</td>
<td>4900</td>
<td>800</td>
<td>230</td>
<td>80</td>
<td>35</td>
<td>19</td>
<td>13</td>
<td>10</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

For perturbations with

$$\frac{k_y^2}{k_\perp^2 k_\parallel^2} > C$$  \hspace{2cm} (3)

the critical current is zero. Every current of an arbitrary intensity leads to the appearance of instabilities.

If condition (3) is not satisfied, the maximal value of $\bar{\xi}_{cr}$ will be $\bar{\xi}_1$. This is always less than the experimentally measured $\bar{\xi}$ values if
W VII A

Temperature and density profiles

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$H_2$

Fig. 1
\[ \frac{k_y}{k_{\perp} k_{\parallel}} > \frac{3}{10} \left( \frac{m_e}{2 m_i} \right)^{1/2} \tau_n \frac{\nu_e Q_i^2}{\Omega_i^2} \]  \quad (4)

It therefore results from the calculations of Ellis and Motley that the current driven drift instabilities cannot be stabilized by the ion viscosity in a collisional plasma if condition (3) or (4) is satisfied.

2) Influence of Toroidal Geometry

An estimate of the influence of the toroidality of the plasma on current driven drift waves in the presence of density gradients in a collisional plasma was made by Horton and Varma \[4]. If \( T_i/T_e > \frac{1}{2} \), a small stabilization is possible. The corresponding shift of the critical \( \xi \) value due to the toroidality is

\[ \Delta \xi_{\text{cor,tor}} \approx \frac{\omega^*}{\tau_n} \frac{\nabla n^2}{\nabla T} \left[ 1 + \frac{T_i}{T_e} \right] \left[ 1 + \frac{\tau_n}{T_i} \left( \frac{T_i}{T_e} \right) \right] \]  \quad (5)

Here we introduced the characteristic parameters of a toroidal plasma. \( t = \frac{R \Phi}{\pi B_\phi} \) with \( B_\phi \) and \( B_\theta \) as toroidal components of the magnetic field. \( \omega^* = -\frac{c k_y T}{n} \frac{\partial n}{\partial x} \) is the drift frequency.

Comparing the calculated \( \Delta \xi_{\text{cor,tor}} \) with the measured \( \xi \) values for the W VII-A stellarators, the values given in Table 2 are found.

<table>
<thead>
<tr>
<th>( r \text{ cm} )</th>
<th>0</th>
<th>1.2</th>
<th>2.4</th>
<th>3.6</th>
<th>4.8</th>
<th>6</th>
<th>7.2</th>
<th>8.4</th>
<th>9.6</th>
<th>10.8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \xi_{\text{cor,tor}} )</td>
<td>27</td>
<td>0.01</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
<td>0.004</td>
<td>0.007</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>

As it can be seen, the shift of \( \xi \) due to the toroidality is negligible in our case.
3) Influence of Simultaneous Temperature and Density Gradients

In the case of an inhomogeneous collisional plasma with density and temperature gradient the growth rate of current driven drift waves depends on the parameter

$$\eta = \frac{\frac{\partial}{\partial \tau} \ln T}{\frac{\partial}{\partial \tau} \ln n} = \frac{r_n}{r_T}$$

Horton and Varma showed that if temperature and density gradients are oriented in the same direction, a stabilizing effect appears. The current driven drift instability can develop only if

$$\xi > \xi_{cr} = \frac{3}{2} \frac{\gamma_e v_d}{v_e} \frac{k_y}{k_{||}}$$

($v_d$ = drift velocity of electrons)

Therefore condition

$$\frac{k_y}{k_{||}} < \frac{2 \xi v_e}{3 \gamma v_d}$$

(6)

limits the possible dimensions of the unstable perturbations.

4. Influence of Shear

In the limit of the slab model shear has a stabilizing effect. Changing the direction of the magnetic field, the $k_{||}$ and $k_{\perp}$ components are changed keeping $k = \text{const}$. This can lead to a transition of the perturbations in the stable region. For the largest scale perturbations admitted by the finite dimensions of the experimental system an estimate of shear stabilization was made by Mikhailovskyi [5]. The following condition results (vol. 2, p. 166):

$$\theta > \left( \frac{\xi v_e \gamma^2}{\Omega} k_{\perp} \right)^{1/3}$$

(7)

(Here we used the following notations: $\theta = \frac{r}{R} |\frac{d\psi}{dr}|$ is the shear and $\gamma_e$ the electron-ion collision frequency). This condition limits the maximal value of $k_{\perp}^{-1}$, which can lead to the appearance of the current convective instability. This value is represented in Fig. 2 by curve a). In the same figure, we
have also represented the minimal values of $k_{\perp}^{-1}$ which result from eqs. (3), (4), and (6). These values correspond to curve b). As an illustration, we represented also the Larmor radius of the ions as a function of $r$ (curve c)). We can see that curve b) lies in the region $k_{\perp} q_i < 1$ of applicability of the fluid theory. Curves a) and b) circumscribe the region in the $(r, 1/k_{\perp})$ plane in which the perturbations lead to instability. In this figure, the stability region is also indicated. In the same figure we have plotted also curve d) for which the growth rate was calculated analytically by Kadomtsev and Pogutse [6] (v. 5, p. 295). In this case, the stability region is given by

$$x_{\perp} \approx \sqrt{\frac{2m_i \xi_i}{m_e T_e} \frac{\xi_i \tau_n}{q_i} \frac{\tau_n v_i}{v_e \tilde{\omega}}}$$ \quad with \quad \tilde{\omega} = \frac{2 \pi r \theta}{\tau_n} \quad (8)$$

The growth rate is

$$\gamma \sim \left[ \frac{\xi_i \tilde{\omega}}{\tau_n} \sqrt{\frac{m_i q_i^2 T_e}{2 m_e T_i}} \right]^{1/2} \frac{v_i}{\tau_n} \quad (9)$$

and the real part of the frequency is

$$\omega \sim \left[ \frac{v_e^2 q_i^3 \xi_i}{T_e \frac{m_i}{m_e}} \left( \frac{T_e}{2 T_i} \right)^{3/2} \right]^{1/4} \frac{v_i}{\tau_n} \quad (10)$$

In Fig. 3 we can see that the appearance of collisional current driven drift instability with $k_{\perp} \gg 0.4$ cm is to be expected for $r > 8$ cm. This is exactly the region of the plasma column in which the mean free path is less than the major radius of the torus and a very high anomalously thermal conductivity appears. The data shown in Fig. 2 are in agreement with the assumption that the thermal conductivity and the anomalous diffusion are produced by collisional current driven drift instabilities.

5) An Estimate of the Instability Region for Collisionless Current Driven Drift Waves

In the case of collisionless current driven drift waves a limitation of $k_y$ (the component of the wave vector in the direction of the $\mathbf{p}$-drift) appears. There are three factors which limit the $k_y^{-1}$ value:
Stability and instability regions in collisional regime

\[ \frac{1}{k_\perp} \text{ [cm]} \]

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STABILITY

INSTABILITY

Fig. 2
a) The shear of the magnetic field lines. The current driven kinetic drift instabilities are stabilized by shear if ([5] vol. 2, p. 165)

\[ \Theta \gtrsim \left( \frac{m_e}{m_i} \right)^{1/2} \left( \frac{q_i}{q_e} \frac{e_i}{e_e} \right)^2 \frac{1}{\ell'_{ky}} \]  

(11)

b) The finite value of the major radius. Using the assumption that \( k_y^{-1} \) must be less than \( R \), we get the condition ([5] vol. 2, p. 165)

\[ k_y^{-1} \lesssim \frac{R \ell'_{ky} e_i}{\tau_n v_i} \]  

(12)

c) The existence of the ion Larmor radius as a limit for the applicability of the two fluid model.

Relations (11), (12), and the dependence of the Larmor radius from \( r \) allows to determine the stability and the instability regions in the \( (r, \frac{1}{k_y}) \) plane. They are represented in Fig. 3. Curve a) corresponds to the inequality (11), curve b) to (12), and c) gives the values of the ion Larmor radius. These three curves determine an instability region for the collisionfree drift waves in the plasma column. This region lies near \( r < 8 \text{ cm} \). It has practical importance only in the vicinity of \( r \sim 6 \div 8 \text{ cm} \). For \( r \sim 6 \div 8 \text{ cm} \), we have a contact between the instability regions of collisional and collisionless instabilities. So we have an instability band around \( k_y^{-1} \sim 0.13 \sim 0.3 \text{ cm} \) in both collisional and collisionless regimes. Therefore, current driven drift instability would be expected to appear in the region \( r > 6 \text{ cm} \).

Conclusions

Our estimates show that the collisional current driven drift waves are important in the region \( r = 6 \div 8 \text{ cm} \) of high thermal conductivity. It is possible to extend our calculations to a variety of experimental situations.
Stability and instability regions in collisionless regime

\[ \frac{1}{k_\perp} \text{[cm]} \]

\[ r \text{[cm]} \]

INSTABILITY

STABILITY

\[ \rho_i \]

Fig. 3
If we compare various situations with different discharges, different temperatures and densities, holding $B$, $\xi$, $\Theta$, and $r_n$ constant, the following conclusions can be made:

For the collisional drift waves we get the proportionalities

$$\left( \frac{1}{k_{\perp}} \right)_{\text{max}} \sim Z n^2 m_i^3 T^{-5/2}$$

$$\left( \frac{1}{k_{\perp}} \right)_{\text{min}} \sim Z^{-3/4} m_i^{3/8} T^{1/2}$$

The relations show that

- if we increase the temperature, the instability region reduces,
- if we increase the density, the instability region increases,
- if we replace the hydrogen with helium, the instability region increases as an influence of the increase of the electric charge of the ions ($Z$).

If we make the same estimates for the collisionless drift waves, we see that the change of $T$ and $Z$ leads only to a shift of the instability region and only the influence of the ion mass leads to a reduction of the dimensions of the instability region.

The shear of the magnetic field has an essential stabilizing influence. A reduction of the dimensions of the instability region for collisional drift waves with the increase of the shear results from (7). For the dependence of the upper limit we obtain $(k_{\perp}^{-1})_{\text{max}} \sim e^{-3}$.

The change of the magnetic field intensity shifts the upper limit $(k_{\perp}^{-1})_{\text{max}}$ proportional to $B^{-2}$ and the lower limit $(k_{\perp}^{-1})_{\text{min}}$ proportional to $B^{-1}$. Therefore, the increase of the magnetic field leads to a reduction of the dimensions of the instability region.
Here, a remark has to be made. Our estimates were made using results which neglect the variation of the plasma parameters along the magnetic field lines in the toroidal system. This effect could lead to an additional increase of the instability regions.

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References


