The Maximum Electron Temperature in the W VII-A Stellarator and Comparison with Tokamak-Discharges

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Abstract

The maximum electron temperature of low density hydrogen discharges in the W VII-A stellarator with ohmic heating is compared with scaling laws derived by Guest, Miller, Pfeiffer, and Waltz. The data show that the maximum temperature can be understood on the basis of an electron heat conductivity $\chi_e$ which scales with $\frac{I(r)}{r^2} \frac{1}{n_e T_e}$ ($I(r) = \text{plasma current within radius } r$). The external rotational transform $\tau_o$ affects the temperature by determining the maximum current density in the plasma center ($j(0) \leq \frac{B}{V_o R} (1 - \tau_o)$, $B =$ magnetic field, $R =$ major radius).

An explicit dependence of $\chi_e$ on $\tau_o$ cannot be derived from the data. A comparison of the W VII-A data with tokamak data shows that the scaling $T_e \sim \left( \frac{B}{R} \right)^2 \frac{a}{R} (1 - \tau_o) (1 - \frac{r^2}{q_1}) \sqrt{Z}$ is a good fit to the maximum electron temperature in ohmically heated discharges rather than $T_e \sim \sqrt{\frac{BR^3/2}{a}} \sqrt{Z}$ as suggested by Guest et al. The condition for the applicability of this scaling law is $(1 - \tau_o) q_a \leq 4 - 5$, in this case the current density in the plasma centre is determined by sawtooth oscillations.
Introduction

In a recent paper G.E. Guest et al. [1] have derived a semi-empirical formula for the electron thermal conductivity in Tokamak discharges. The authors suggest that the electron thermal conductivity is described by

\[
\chi_e = \text{Const} \frac{a}{R^{3/2}} \frac{I(r)}{r^2 n_e(r) \sqrt{T_e}}
\]  

(1)

\(R = \) major radius of the device, \(a = \) minor radius of the plasma, \(B = \) toroidal magnetic field, \(I(r) = \) plasma current within a cylinder with radius \(r\), \(n_e T_e = \) density, temperature of electrons.

A similar formula was derived before by Coppi et al. [2] in order to explain ALCATOR results. The factor \(\frac{a}{R^{3/2}}\) is derived by dimensional analysis according to a paper of Connor and Taylor [3]. For a given discharge \(\frac{a}{R^{3/2}}\) is a constant and does not affect the shape of the plasma profiles. The dependence on \(B\) can be checked in one experimental device but the existence of the factor \(\frac{a}{R^{3/2}}\) can be proven only by comparison of different machines.

If the radiation losses and ion heating are neglected, the heat balance equation of the electrons can be solved explicitly, the temperature profile is proportional to \([1 - \left(\frac{r}{a}\right)^2]^{1/2}\) and the maximum electron temperature is given by the formula

\[
T_e(\circ) = I \left[ \frac{1}{2\pi} \frac{Z_{\text{eff}} \hat{\eta}}{f_0 g(R, a)} \frac{\mu_o R}{2\pi} \frac{1}{a^2} \right]^{1/2} [q_a \frac{q_{a-1}}{3}]^{1/2}
\]

(2)

\[
\hat{\eta} = \frac{e^2 c^2 \sqrt{m_e}}{1.16} \frac{3}{2 \pi} , \quad q_a = q \text{ on the boundary, } f_0 = \text{const},
\]

\(q = \frac{a}{R^{3/2}}\). This formula holds for \(q_a \leq 4\). In this regime
a q=1 domain exists in the plasma centre and sawtooth oscillations limit the current density to

\[ J_{\text{max}}(o) = \frac{2B}{\mu_o R} \]

In a stellarator with ohmic heating the maximum current density is given by \( J_{\text{max}}(o) = \frac{2B}{\mu_o R(1-\xi_o)} \), where \( \xi_o \) is the external rotational transform from the helical windings.

Also the radius of the q=1 surface is modified by the external transform. Following the procedure of ref.[1] the electron energy balance equation can be solved taking into account the external transform \( \xi_o \). \( \xi_o \) is assumed to be constant over the plasma radius, therefore this model describes a shearless l=2 stellarator with ohmic heating.

The result of the calculation is

\[ T_e(o) = \text{Const} \left( \frac{B}{g(R,a)} \right) ^{1/2} \left( \frac{a^2 Z_{\text{eff}} B}{R(1-\xi_o)} \right) ^{1/2} \left( \frac{q_a - 1}{3q_a} \right) ^{1/2} \] \hspace{1cm} (3)

The Constant const includes \( \eta \), \( f_e \) and numerical factors.

In the case \( \xi_o \rightarrow o \) formula (3) is reduced to formula (2).

The effect of the external transform is twofold: the maximum current density in the centre is reduced by the factor \( 1-\xi_o \).

The radius of the q=1 surface is increased which also leads to a reduction of the central temperature. With increasing plasma density radiation losses \[4\] and heating of the ions \[5\] play a dominant role in determining the maximum electron temperature, therefore formula (3) yields an upper limit for the temperature. In tokamak devices most data are taken around \( q_a = 4 \), in this regime the effect of sawtooth oscillations is not yet important. If we neglect the reduction of the effective plasma radius by sawtooth oscillations, the maximum temperature is

\[ T_e(o) \leq \text{Const} \left( \frac{B}{g(R,a)} \right) ^{1/2} \left( \frac{a^2 Z_{\text{eff}} B}{R(1-\xi_o)} \right) ^{1/2} \] \hspace{1cm} (4)
Experimental Results

The formula (4) predicts the maximum temperature being independent of the plasma current. This effect is shown in 3 helium discharges in W VII-A with \( B = 3.3 \) T, \( t_o = 0.17 \) (Fig. 1). The current is varied between 13 and 34 kA, \( n_e(0) = 5 \times 10^{13} \text{cm}^{-3} \) in all three cases. The \( q=1 \) radius as calculated from the temperature profiles (\( Z_{\text{eff}}=\text{const} \)) expands from \( r_{q=1} = 2 \) cm to \( r_{q=1} = 6 \) cm (Fig. 2). The central temperature even decays at high current indicating the effect of the shrinking confinement zone. At 34 kA \( q(a) = 2 \) and the region of sawtooth oscillation extends to more than 50% of the plasma radius. This was also confirmed by measurements of the X-ray diodes (Fig. 3).

The W VII-A data are taken from the hydrogen discharges [6],[7], the density regime is \( n_e(0) = 1 - 5 \times 10^{13} \text{cm}^{-3} \). In this regime radiation losses or heating of ions are not dominating. The maximum temperature is plotted versus

\[
Z = Z_{\text{eff}} \frac{B}{R}(1-t_o) \left( \frac{a}{0.11} \right)^4 \left( 1 - \frac{t_p}{1-t_o} \right),
\]

the magnetic field was varied between 2.5 T and 3.5 T, \( t_o = 0.055 - 0.23 \). The case \( t_o = 0.055 \) is comparable with tokamak discharges since the transform \( t_o \) is much smaller than the transform of the heating current.

The data of W VII-A show a linear increase of the central temperature \( T_e(0) \) with

\[
\sqrt{Z_{\text{eff}} \frac{B}{R}(1-t_o)^4 \left( 1 - \frac{t_p(a)}{1-t_o} \right)} \quad (\text{Fig. 5})
\]

the optimum can roughly be described by

\[
T_e,\text{max} = 360 \sqrt{Z_{\text{eff}} \frac{B[T]}{R[m]}(1-t_o)^4 \left( 1 - \frac{t_p(a)}{1-t_o} \right)} \quad [\text{eV}],
\]

the bulk of the data follows the line \( T_e = 290 \sqrt{Z} \quad [\text{eV}] \)
The maximum deviation is 25% which might be caused by the neglected effects: radiation, ion heating and the shrinkage of the confinement zone due to changes of the profile.

Therefore, the conclusion can be drawn that the maximum temperature in W VII-A roughly scales with $Z_{\text{eff}} \frac{B}{R}(1-t_o)$, whether the factor $\sqrt{B}$ (see eq. (4)) exists or not cannot be decided since the variation in B is too small. It also cannot be decided whether the thermal conductivity $\chi_e$ explicitly depends on the external helical field. If there is any dependence of $\chi_e$ on $t_o$, this dependence must be small, for $t_o \leq 0.23$ the variation of $\chi_e$ with $t_o$ cannot be more than 50%, otherwise the maximum temperature would be outside the experimental data. The total transform $t_P + t_o$ has been varied only by a factor of 2, therefore a strong variation of the confinement with $t_o$ cannot be expected.

The radius of the q=1 surface is derived neglecting the effect of radiation on the temperature and current profiles. A shrinkage of the current channel due to increasing radiation leads to a steepening of the current profile and an increase of the q=1 regime. Therefore, the reduction factor $\frac{4}{3}(1-\frac{t_P}{1-t_o})$ calculated above is too large, sawtooth oscillations and a q=1 surface already occurs for $\frac{t_P}{1-t_o} < \frac{1}{4}$ in many devices. In the calculations above the effect of a radiation layer and the shrinkage of the current channel can be simulated by a reduced plasma radius $a = a(1-\Lambda)$, $\Lambda$ is the width of the radiation layer.

Fig. 4 shows the width of the confinement layer $a - r_{q=1}$ vs. $\frac{t_P(a)}{1-t_o}$, the experimental data in Fig. 4 are taken from helium discharges in W VII-A. It can be seen that especially at low plasma currents the radius $r_{q=1}$ is larger than calculated in this simple model.
Comparison With Tokamak Discharges

In order to check equation (3) data are collected from several tokamak experiments (PULSATOR, ALCATOR, TFR, ST, PLT, Frascati Tokamak, T-10). Most of the data are taken from a collection by Pfeiffer and Waltz [8] and from the results reported at the Berchtesgaden and Innsbruck Conferences [9], [10], [11], [12], [13].

The maximum temperature is plotted versus $Z$, the factor $\sqrt{B/g(R,a)}$ is not taken into account in Fig. 6. Since the tokamak data are mainly taken at $q(a) > 4$ ($\epsilon_p < 0.25$) only some PULSATOR data have $q(a) = 3.5$, the reduction factor due to sawtooth oscillations is set to 1 in eq. (3). The data also include the high density discharges, therefore it has to be expected that especially in this case $T_e < 360 \sqrt{Z}$ [eV].

Fig. 6 shows that the maximum temperature in the ohmically heated devices can be described by $360 \sqrt{Z}$ [eV]. Close to the high density limit (PULSATOR, ALCATOR), the deviation from this line is maximum. In order to check the existence of the factor $\sqrt{B/g(R,a)}$, the normalized temperature $T_e \sqrt{g(R,a)/B}$ is plotted versus $Z$ (Fig. 7).

A comparison of Fig. 6 with Fig. 7 shows that the scaling $T_e \sim \sqrt{Z}$ is a better fit to the experimental data than $\sqrt{B/g(R,a)} \cdot \sqrt{Z}$. The difference between the two figures is not very large, but there are some facts which are in favour of the $T_e \sim \sqrt{Z}$ scaling. In Fig. 7 the ISX data are above the optimum line of the other tokamak data, but as it was pointed out in the ISX paper [5] the heating of ions already plays an important role in the ISX device. Therefore, it should be concluded that either ISX and W VII-A follow a scaling of the thermal conductivity of the electrons different from other tokamaks, or - as shown in Fig. 6 - the ohmically heated discharge is described by $T_e \sim \sqrt{Z}$. But this means that the factor $\sqrt{B/g(R,a)}$ does not exist.
Discussion

The scaling law $T_e(0) \sim \sqrt{z}$ as confirmed by the experimental data does not necessarily confirm the radial dependence of the thermal conductivity $\chi_e \sim \frac{I(r)}{r^n \sqrt{T_e}}$. Any function $\chi_e(r) \sim \frac{I(r)}{r^n} g(r)$, where $g(r)$ is a normalized function $\int_0^r g(r) \, dr = 1$ describing the localization of fluctuations which give rise to the anomalous conductivity, can lead to the same scaling laws (within a constant factor). If $g(r)$ is independent of the plasma current, temperature and density, the scaling law of $T_e(0)$ is also independent of $g(r)$.

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References


Figure Captions

Fig. 1: Electron temperature profile of three helium discharges with different current ($B = 3.3\ T, \ \epsilon_o = 0.17$).

Fig. 2: Profile of the rotational transform ($\epsilon(r)$) under the same conditions as in Fig. 1.

Fig. 3: Intensity of sawtooth oscillations measured by X-ray diodes. Comparison with the electron temperature profile.

Fig. 4: Radius of the confinement zone $a_{\text{q}=1}$ as $\epsilon_P(a)$ a function of $\frac{P}{1 - \epsilon_o}$. The experimental data are derived from the $T_e(r)$ profiles and the X-ray measurements (see Fig. 3).

Fig. 5: Maximum electron temperature $T_e(0)$ vs.

$$Z_{\text{eff}} \frac{B}{R} (1 - \epsilon_0) \frac{4}{3} (1 - \frac{\epsilon_P(a)}{1 - \epsilon_0})$$

data are from hydrogen discharges in W VII-A.

Fig. 6: Maximum electron temperature $T_e(0)$ vs.

$$\left(\frac{a}{0.1} \right)^2 \frac{B}{R} (1 - \epsilon_0) \frac{4}{3} (1 - \frac{\epsilon_P(a)}{1 - \epsilon_0})$$

for second ohmically heated discharges. Since most of the tokamak data are from the regime $q_a = 4 - 6$, the reduction factor $\frac{4}{3} (1 - \frac{\epsilon_P(a)}{1 - \epsilon_0})$ is set to 1 in this case.

Fig. 7: The normalized temperature $T_e(0)\sqrt{\frac{a}{BR^{3/2}}} = T^* \text{ vs. } Z$. 
Fig. 1

Fig. 2
Fig. 3

Fig. 4
Maximum Electron Temperature

$T_e(0) = 360\sqrt{Z}$

$T_e = 290\sqrt{Z}$

**W VII A**

$H_2$

- $t_0 = 0.23$
- $t_0 = 0.11$
- $t_0 = 0.055$

$Z = Z_{eff} \frac{B(T)}{R[m]} \left(1 - t_0\right) \frac{4}{3} \left(1 - \frac{t_p(a)}{1 - t_0(0.11)} \right)$
Maximum Electron Temperature of Ohmically Heated Discharges.

\[ Z =: \left( \frac{a}{0.11} \right)^2 Z_{\text{eff}} \frac{B}{R} \left( 1 - t_0 \right) \frac{4}{3} \left( 1 - \frac{t_p(a)}{1 - t_0} \right) \]
\[ T_e = T_e(0) \sqrt{\frac{g(R,a)}{B}} \]

\[ Z = \left( \frac{a}{0.11} \right)^2 Z_{\text{eff}} \frac{B}{R} \left( 1 - \tau_0 \right) \frac{4}{3} \left( \frac{1 - \frac{t_p(a)}{1 - \tau_0}}{1 - \tau_0} \right) \]