Electron Ring Diagnostics
with Light Scattering

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Abstract

The scattering of light from the relativistic electrons in an electron ring can be used to determine the radial (and axial) betatron amplitude distribution and the spread of the closed orbit radii of the electrons (as given by the electron energy spread) separately. The betatron oscillation amplitudes are obtained from the angular scattering distribution, and the energy spread from the shape of the spectrum of the scattered light. Certain characteristics of the scattering spectrum depend only on the standard deviation of the electron energy distribution, but they are relatively insensitive to the distribution function.
1. Introduction

The effectiveness of a collective ion accelerator with relativistic electron rings is mainly given by the magnitude of the holding power, the maximum electric field strength in the accelerated rings. This quantity can be brought to the desired high values not only by increasing the electron number in the rings but also by lowering the major and minor ring dimensions. While the major electron ring radius is only determined by the magnetic compression process, the minor dimensions are moreover influenced by the initial conditions of the ring, as the injection and inflection mechanisms, as well as by resonances and collective instabilities.

The axial density profile of an electron ring is only given by the distribution of the axial betatron oscillation amplitudes, while the radial density distribution is determined by the radial betatron oscillation amplitude distribution and by the spread of the closed orbit radii as given by the electron energy spread.

These both quantities, the energy spread and the betatron oscillation amplitude distribution, however, are not only established by the initial conditions, but may also be changed during the process of ring compression due to betatron resonances and collective instabilities. Therefore it is very important to know the amounts with which the betatron oscillations or the momentum spread of the electrons contribute to the radial minor dimension, in order to find out which mechanism (betatron resonance or collective longitudinal instability) leads to the increase of the minor radial electron ring dimension which results in a lowering of the holding power.

The radial and axial density distributions can be measured from the X-ray emission at thin wires with which the ring is scraped during its motion in radial or axial direction. As this method normally leads to a change of the ring dimensions due to its interference with the scraper, it should only be used for a rough information. Therefore the use of the synchrotron radiation is
preferred, with which the ring minor cross section can be imaged $^{1,2}$. This method, however, does not distinguish between the spread in electron momentum or the spread in the radial betatron oscillation amplitude.

Peterson and Rechen$^3$ proposed a method of analyzing the momentum and betatron oscillation amplitude distributions in a circulating beam by a two-probe method, where one probe clips the beam at the inner radius, and the other at the outer radius. The distributions in particle momentum and in amplitude of radial betatron oscillations are obtained from the gradient function of the X-ray signals. This method has been applied in the Berkeley electron ring experiments $^4$.

In this report we propose a different method to measure the particle momentum and radial betatron oscillation distribution, the interference of which with the electron ring is almost negligible. It takes advantage of the energy sensitivity and the narrow angular distribution of the light scattered at relativistic electrons. Moreover it gives the possibility to measure the distributions in particle momentum and in radial betatron oscillation amplitude separately: the betatron oscillation distribution from the angular radiation distribution of the scattered light and the particle momentum spread from the scattering spectrum.

2. Scattering of laser light by relativistic electrons

2.1 Scattering cross sections

Laser scattering as a diagnostic tool has often been proposed and applied to plasmas (see e.g. ref.5). The first results of laser light scattered by an electron beam (of non-relativistic velocity) were obtained by Fiocco and Thomson $^6$. After calculations of the relativistic Thomson scattering cross section by Pechacek and Trivelpiece $^7,8$, Ward $^9,10$ studied experimentally and theoretically the scattering of laser light by 50 keV electrons in a beam. The scattering from electrons in the University of Maryland relativistic plasma column $^{11}$ has not been successful
yet\(^{12}\). The laser light scattered at highly energetic electrons
to produce beams of hard polarized \(\gamma\) rays has been proposed by
Arutyunyan et al.\(^ {13,14}\) and by Milburn\(^ {15}\) already in 1963. Some
years later these polarized photon beams were produced at the
Stanford linear accelerator (SLAC)\(^ {16}\) and at the Cambridge Elec-
tron Accelerator (CEA)\(^ {17}\).

In the following we restrict ourselves to the scattering of mono-
chromatic linearly polarized light on free electrons.

The differential cross section \(d\sigma\) for the scattering of a plane
monochromatic wave, which is linearly polarized and which energy
is negligible compared to that of the electron, at a free elec-
tron at rest is given by\(^ {18}\)

\[
d\sigma = \left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2 \sin^2 \tilde{\psi} \, d\tilde{\sigma},
\]

where \(\tilde{\psi}\) is the angle between the direction of the wavevector of
the scattered light and the incident electric vector. The solid
angle \(d\tilde{\sigma}\) is given by

\[
d\tilde{\sigma} = \sin \tilde{\psi} \, d\tilde{\psi} \, d\varphi.
\]

By integrating over \(\tilde{\psi}\) from 0 to \(\pi\) and over \(\varphi\) from 0 to \(2\pi\) we ob-
tain the well-known Thomson formula

\[
\sigma_{\text{Thomson}} = \frac{8\pi}{3} \left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2 = \frac{8\pi}{3} r_e^2,
\]

\(r_e\) being the classical electron radius \(r_e = \frac{e^2}{4\pi\varepsilon_0 mc^2} = 2.818 \times 10^{-13}\) cm
such that \(\sigma_{\text{Thomson}} = 6.65 \times 10^{-25}\) cm\(^2\) = 665 mb. The frequency of
the scattered light is the same as that of the incoming wave.

In the case, in which the photon energy is no more negligible
compared to the electron (Compton effect), as well as in the
case, where the electron is not at rest initially, the frequency
of the scattered light differs from that of the incoming light.

The Compton effect is extensively treated in the literature\(^ {13-19}\).
In this report we only use the expressions for the scattering of
light, the energy of which is very small compared to the electron.
energy, since this case is of interest for the diagnostics of relativistic electron rings by light scattering with present day lasers.

If a photon with frequency $\omega_0$ and wave vector $\mathbf{k}_0$ hits an electron of momentum vector $\mathbf{p}$ at an angle $\theta$ (Fig.1), and the polarization vector is perpendicular to $\mathbf{p}$,* the frequency of the scattered radiation is given in the laboratory system by

$$\omega_1 = \omega_0 \frac{1 + \beta \cos \theta}{1 - \beta \cos \chi},$$

where $\beta$ is the electron velocity relative to the speed of light $c$, and with it $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic factor of the electron. $\chi$ is the angle between the wave vector $\mathbf{k}$ of the scattered photon to the direction $\mathbf{p}$ of the incoming electron.

We always assume the photon energies $\hbar \omega_0$ and $\hbar \omega_1$ to be negligible compared to the electron energy $\gamma m_0 c^2$, which is very well fulfilled for ruby laser light and non-ultrarelativistic electrons.

* We choose this direction of polarization, since this case is suitable for the application envisaged. The results for the other direction of polarization are quite similar.
For head-on collision of the photon and the electron ($\Theta = 0$) the maximum energy $\omega_1$ of the scattered photons (for $\chi = 0$), in the direction of the electron, is by a factor of

$$\frac{\omega_1}{\omega_0} = \frac{1 + \beta}{1 - \beta} = \frac{\gamma^2 (1+\beta)^2}{(1-\beta)\gamma^2} \approx 4 \gamma^2 \text{ for } \gamma \gg 1$$

higher than the energy of the incident light. The scattered photons from ruby laser light ($\lambda_o = 6943 \text{ Å}$) on $\gamma = 30$ electrons, for example (that are typical in electron ring accelerators), may be found in the 2 Ångström wavelength region.

The differential scattering cross section for the photons (in the laboratory frame) is found by several authors, taking the effect of the "finite" scattering region into account $7,9,10,19-24$

$$d\sigma(\chi, \phi)_{ph} = \frac{r_e^2}{\delta^4} \left( \frac{1 + \beta \cos \Theta}{1 - \beta \cos \chi} \right)^4 \left( \frac{(\beta - \cos \chi) \delta^2 + \sin^2 \phi \sin^2 \chi}{\delta^4 (1 - \beta \cos \chi)^2} \right) d\Omega =$$

$$= r_e^2 (1 + \beta \cos \Theta) \frac{1}{\delta^4 (1 - \beta \cos \chi)^2} \left[ \frac{\sin^2 \chi \cdot \cos^2 \phi}{\delta^4 (1 - \beta \cos \chi)^4} \right] d\Omega$$

with $d\Omega = \sin \chi d\chi d\phi$.

This scattering cross section is a relativistic invariant in the case of $\Theta = \pi/2$. Thus the integration over $\phi$ (for $\Theta = \pi/2$) gives:

$$d\Sigma(\chi)_{ph} = \int d\sigma(\chi, \phi)_{ph} d\phi = \frac{\pi r_e^2}{\delta^4} \left[ \frac{2 \delta^2}{(1 - \beta \cos \chi)^2} - \frac{\sin^2 \chi}{(1 - \beta \cos \chi)^4} \right] \sin \chi d\chi$$

and the further integration over $\chi$ results in the Thomson formula (equ. 2) $^9$

$$\bar{\Sigma}_{ph} = \int d\Sigma(\chi) d\chi = \frac{8 \pi}{3} \frac{r_e^2}{\delta^2} = \bar{\Sigma}_{\text{Thomson}}.$$
Multiplying by a factor \( \frac{1 + \beta \cos \Theta}{1 - \beta \cos \lambda} \) allows to convert from the differential photon scattering cross section to an energy scattering cross section:

\[
\begin{align*}
\frac{\Delta \sigma(\chi, \phi)}{\Delta \omega} &= \frac{e^2}{\hbar^2} \frac{(1 + \beta \cos \Theta)^2}{(1 - \beta \cos \lambda)^5} \left[ (\beta - \cos \chi)^2 + \sin^2 \phi \cdot \sin^2 \chi \right] d\omega,
\end{align*}
\]

with \( d\omega = \sin \chi d\chi d\phi \).

Integration of this differential cross section over \( \phi \) results in

\[
\Delta \Sigma(\chi)_{\text{en}} = \frac{\pi e^2 (1 + \beta \cos \Theta)^2}{\hbar^2} \left[ \frac{2 \beta^2}{(1 - \beta \cos \chi)^3} - \frac{\sin^2 \chi}{(1 - \beta \cos \chi)^5} \right] \sin \chi d\chi,
\]

and the further integration over \( \chi \) gives

\[
\begin{align*}
\sigma_{\text{en}} &= \int_0^\pi \Delta \Sigma(\chi) d\chi = \frac{8\pi}{3} \frac{e^2}{\hbar^2} (1 + \beta \cos \Theta)^2 \xi \text{m_{e.m.}} \xi^2 (1 + \beta \cos \Theta)^2,
\end{align*}
\]

which expressions agree with those given by Ward \(^9\) (in a different notation) that include the effects of a finite interaction region \(^7\)\(^-\)\(^9\). The expressions are sums over all polarization directions of the scattered photons. The intensity measurements of certain directions of polarization of the scattered photons might give some further information of the electrons in the relativistic electron rings, which at first, however, will not be used.

For \( \beta \neq 0 \) the scattering cross sections \( \Delta \sigma(\chi, \phi)_{\text{ph}} \) and \( \Delta \sigma(\chi, \phi)_{\text{en}} \) have a very strong angular dependence, that peaks in forward direction (\( \chi = 0 \)) and forms a narrow cone around this direction. The cross sections for \( \phi = 0 \) get zero at an angle of

\[
\chi_0 = \arccos \beta = \arcsin \frac{1}{\xi} \quad (\approx \frac{1}{\xi} \text{ for } \xi \gg 1).
\]

For \( \gamma = 27 \) the angle is as small as \( \chi_0 = 2.1225^\circ \). For a value of \( \gamma = 1.2 \) we get \( \chi = 56.44^\circ \). This \( \gamma = 1.2 \) case is chosen for a demonstration of the angular dependence of the scattering cross
Fig. 2 An example of the angular dependence of the differential energy scattering cross section for the case of $\gamma = 1.2$
section \(d\sigma(\chi, \phi)_{en}/d\Omega\) normalized to \(r_e^2(1+\beta\cos\theta)^2/\gamma^4\) in figure 2 (in polar coordinates). Even for this very small value of \(\gamma\) we obtain a very pronounced peak of the cross section distribution in forward direction.

Half of the photon number is scattered into the cone with an aperture angle of \(\chi_0\) around the forward direction, as follows from integrating \(d\Sigma(\chi)_{ph}\) in equ.(5) over \(\chi\) from 0 to \(\chi_0 = \arccos \frac{1}{\gamma}\) (for \(\theta = \pi/2\)):

\[
\int_0^{\chi_0} d\Sigma(\chi)_{ph} d\chi = \frac{4\pi}{3} r_e^2 = \frac{1}{2} \sigma_{Thomson}.
\]

2.2 Estimate of the scattered photon number

The total scattered photon number \(N_{ph}\) is obtained from

\[
N_{ph} = N_p \cdot n_e \cdot L \cdot \sigma_{ph},
\]

where \(N_p\) is the number of incident photons \(N_p = U_{Laser}/h\nu\), as the ratio of Laser energy \(U_{Laser}\) and single incident photon energy \(h\nu\), \(n_e\) is the electron density in the ring, \(L\) is the interaction length and \(\sigma_{ph}\) is the scattering cross section (equ.(6)). We always assume incoherent scattering. With an energy \(U_{Laser} = 3\) J of a ruby Laser (\(h\nu = 1.785\) eV = \(2.859 \cdot 10^{-19}\) J) we obtain a number \(N_p = 1.05 \cdot 10^{19}\) of incident photons. For moderate electron ring conditions we have an electron density of \(n_e = 1.6 \cdot 10^{11}\) cm\(^{-3}\) and an interaction length of about \(L = 0.5\) cm, so that the total number of scattered photons is \(N_{ph} \approx 5.6 \cdot 10^5\).

This is a relatively high number, which should enable us to diagnose distributions in energy and/or angle, moreover since the scattering occurs in a very narrow angle, which can be estimated from the number of \(N_{ph} \approx 1.5 \cdot 10^5\) that are already found in a cone of only \(1\)° aperture angle for \(\gamma = 27\) electrons assuming negligible electron and Laser emittance.
3. Diagnostic application of light scattering

The characteristic angular distribution (equ. (4)) and the frequency of the scattered light (equ. (3)) open a wide area of applications:

a) The method can be used to determine the radial or axial electron density distribution by scanning the electron ring in radial or axial direction with the strongly focused laser beam and collecting the scattered photons in a relatively large aperture angle (compared to $\chi_0$) around the mean electron direction.

b) If the angular spread of the electrons in radial or in axial direction is equal to or larger than $\chi_0$, the evaluation of the angular intensity distributions of the scattered light allows to determine the radial or axial velocity distributions of the electrons, from which the corresponding betatron oscillation amplitude distributions can easily be obtained. The axial electron ring width is only determined by the axial betatron oscillation amplitude distribution, which thus can be obtained also from other measurements (e.g. synchrotron radiation profile measurements$^2$). The radial betatron oscillation amplitude distribution, however, is of specific interest, since the radial ring width is composed of the contributions of betatron oscillation amplitude distribution and closed orbit distribution due to the electron energy spread. In the following we will therefore concentrate on the angular distribution of scattered light as given by the radial electron velocity distribution. The axial electron velocity will qualitatively give the same results.

c) Since the frequency of the scattered light is strongly depending on the electron energy (equ. (3)), the measurement of the spectrum gives information about the electron energy distribution and hence the closed orbit spread in the ring. The evaluation of the closed orbit distribution of the electrons in the ring helps to solve the question, if during the process of compression the negative mass instability has occurred and has thus lead to energy spreading or not.
3.1 Angular scattering energy distribution

The differential energy scattering cross section is given by equ.(7). If we have an electron distribution function $f_e(r, z, \psi, \zeta)$ in the electron ring, where $\psi$ is the angle in radial $(r)$ and $\zeta$ that in axial direction $(z)$ (see fig.3), the differential scattering cross section (equ.(7)) has to be expressed with these angles:

$$\frac{d\sigma(\psi, \zeta)_{en}}{d\Omega} = \frac{r_e^2(1 + \beta \cos \Theta)^2}{\delta^2(1 - \beta \cos \psi \cos \zeta)^3} \left[ 1 - \frac{\sin^2 \psi}{\delta^2(1 - \beta \cos \psi \cos \zeta)^2} \right]$$

with $d\Omega = \cos \psi d\psi d\zeta$

and since the time-averaged power scattered per unit solid angle by a single electron $\frac{de(\psi, \zeta)}{d\Omega}$ is given by

$$\frac{de(\psi, \zeta)}{d\Omega} = S \cdot \frac{d\sigma(\psi, \zeta)_{en}}{d\Omega}$$

and

$$|S| = \sqrt{\frac{e_0}{\mu_0}} |E_0|^2$$

being the amount of the Poynting vector of the incident light wave, we have

$$\frac{de(\psi, \zeta)}{d\Omega} = \sqrt{\frac{e_0}{\mu_0}} |E_0|^2 \frac{r_e^2(1 + \beta \cos \Theta)^2}{\delta^2(1 - \beta \cos \psi \cos \zeta)^3} \left[ 1 - \frac{\sin^2 \psi}{\delta^2(1 - \beta \cos \psi \cos \zeta)^2} \right].$$

If we now are interested in the radial angular distribution of the scattered energy, we could think of a detector that integrates the scattered energy over $\zeta$ (in axial direction). The $\psi$-dependence $W(r, z, \psi)$ is then given by

$$W(r, z, \psi) = \int_{-\pi}^{+\pi} \int_{-\pi/2}^{+\pi/2} \int_{-\pi/2}^{+\pi/2} f_e(r, z, \psi, \zeta) \frac{de(\psi, \zeta)}{d\Omega} \cos(\psi - \psi_1) d\psi_1 d\zeta d\phi.$$
Fig. 4 Angular scattering energy distributions for different betatron angular widths
Two typical radial angular distributions \( W(\psi) \), normalized to
\[
\frac{1}{2\pi} \frac{\epsilon_0}{M_0} E_0 \left| \frac{r_2}{\delta^4} \right|^2 (1 + \beta \cos \Theta)^2 \int_{-\delta}^{\delta} \int_{-\delta/2}^{\delta/2} f_e(r, z, \psi) \cos \psi \, dr \, d\psi
\]
are plotted in fig. 4, for a rectangular radial betatron oscillation angular distribution \( f_e(\psi) \) in a), and for a corresponding parabolic distribution in b). We assumed \( \gamma = 27, \beta = 0.9993139 \).

The plotted curves show a strong influence of the betatron oscillation angular distribution on the observed angular scattering energy distribution besides for those narrow spreads being smaller than \( \pm 1^\circ \). The dependence of the profile full width \( \Delta \psi \) at half maximum (FWHM) is plotted in fig. 5 over the angular standard deviation \( \tilde{\sigma}_\psi \) showing relatively good agreement of the rectangular and parabolic betatron oscillation distribution and moreover the expected linear relationship of \( \Delta \psi \) to \( \tilde{\sigma}_\psi \) for standard deviations larger than about \( 1^\circ \).

The width of the radial betatron oscillation angular distribution hence is to be obtained relatively accurately from the measured width of the angular scattering energy distribution. From this consequently the betatron oscillation amplitude distribution can be calculated, and if the total radial minor density distribution is known (as described before or from synchrotron radiation measurements), the relation between the contribution of betatron and synchrotron amplitudes to the minor radial dimension can be estimated. The synchrotron amplitude (closed orbit) distribution (or the electron energy distribution), can be directly obtained from the spectrum, as described in the next chapter.

Before this we should mention that up to here we have operated with the scattering energy distribution without regard of spectral resolution. With frequency dependent detectors we will get slightly different profiles, however, only in the region where we have the deviation from linearity anyway. If we would take out a narrow frequency interval and if we had negligible energy spread of the electrons we even could extend the linear region in fig. 5 to smaller values of \( \tilde{\sigma}_\psi \).
Fig. 5 Angular scattering profile width versus the radial angular standard deviation of the betatron oscillation distribution.

For typical ERA experiments $^{1,2,25}$ one is in the linear regime of fig.5, as might demonstrate the example of a maximum radial betatron oscillation amplitude of 0.3 cm at a major radius of 2.3 cm during the maximum compression of the electron ring, where $\nu_r \approx 1$, so that $\psi_{max} \approx \arctan \frac{0.3}{2.3} \approx 7.5^\circ$, which for a parabolic angular distribution corresponds to $\tilde{\sigma}_\psi = 3.3^\circ$. 
3.2 Spectral distribution of the scattered light

If we again assume incoherent light scattering, the intensity \( P(\omega) \), i.e. the scattered power per unit frequency interval \( d\omega \) and per unit solid angle \( d\Omega \), for an electron distribution function \( f_e(r,z,\chi,\phi) \) is given by

\[
P(\omega) = \iint \frac{d^2p(\chi,\phi,\omega)}{d\omega d\Omega} f_e(r,z,\chi,\phi) drdz \sin\chi d\chi d\phi,
\]

where \( \frac{d^2p(\chi,\phi,\omega)}{d\omega d\Omega} \) is the power scattered per unit frequency interval into the unit solid angle by a single electron, as obtained from equ.(3) and (7), and according to equ.(13)

\[
\frac{d^2p(\chi,\phi,\omega)}{d\omega d\Omega} = \sqrt{\frac{\varepsilon_0}{\mu_0}} |E_0| E_0^2 \frac{r_e^2 (1+\beta \cos\Theta)^2}{\delta^4 (1-\beta \cos\chi)^5} \left[(\beta - \cos\chi)^2 + \sin^2\phi \sin^2\chi\right].
\]

and

\[
\omega_1 = \omega_0 \frac{1+\beta \cos\Theta}{1-\beta \cos\chi}.
\]

In the specific case of the plane geometry, where the propagation vector of the incoming and of the outgoing radiation and the velocity vector of the electrons lie all three in the same plane perpendicular to the polarization vector, i.e. in the case of \( \phi = \frac{\pi}{2} \), TSAKIRIS et al. calculated the spectra of four different particle distribution functions for an observation angle of \( \zeta = \pi/4 \), which is of interest for the electromagnetic wave scattering from the Maryland relativistic rotating plasma column. In contrast to this, in electron rings we have to take the scattering into the total solid angle into account (also for \( \phi \neq \frac{\pi}{2} \)), especially since due to the electron betatron oscillations in typical electron rings we get contributions from all directions into our detecting system.
Therefore we decided to open the angle of detection such that (in principle) the total spectrum integrated over the solid angle is observed. Since for $\gamma = 27$ electrons the main portion of the scattered power (especially the high frequency part of the spectrum) is within the characteristic angle of $\chi_0 \approx 2.1^\circ$ (see equ.(10)), this means in practice that the detector should cover a solid angle of at least $\chi_0$ plus the expected maximum angle that the electrons enclose with the azimuthal direction, which easily can be obtained.

Since the differential scattering cross section (equ. 7) depends on the angle $\Theta$ between the incoming electron and the incident photon, the spectrum will depend on the angular distribution of the incoming electron beam and the incident Laser beam.

If for simplicity we assume a homogeneous distribution of the angles $\Theta$ of the width $2\Theta_o$ around the average angle $\Theta_1$ between monoenergetic electrons and monochromatic incident light,

$$\Theta_1 - \Theta_0 \leq \Theta \leq \Theta_1 + \Theta_0,$$

the spectral distribution of the power radiated into $4\pi$ (normalized to $4\pi \Theta_1^2$ and to $\int e(r,z,\chi,\phi)\,dr\,dz$ sin $\chi\,d\chi\,d\phi$) is plotted in figures 6a to d for different mean angles of $\Theta_1$ with $\Theta_o$ as a parameter. It is obvious that for large angles $\Theta_1$ the spectrum is very sensitive to a spread in $\Theta_o$, while for small angles $\Theta_1$ the spectrum practically does not depend on $\Theta_o$.

Since one wants to get the information of the electron energy distribution from the shape of the spectrum, in electron rings, however, there is a spread of angles $\Theta_0$ between the electrons and the incoming photons in the indicated range, one has to work at a mean angle of $\Theta_1 \approx 0$, i.e. with incident photons hitting the electrons nearly head-on.

We now fix the angle $\Theta_1$ to be about zero and calculate the spectrum of the power radiated into $4\pi$ for different energy distribution functions of the electrons. We always assume monochromatic incident (laser) light. In defining the standard deviation $\sigma_\gamma$ for a certain width and shape $f(\gamma)$ of the distribution functions with
Fig. 6a and b Spectral distribution of monoenergetic electrons for different spreads and mean values of the angle between the incident photons and the electrons.
Fig. 6c and d: Spectral distribution of monoenergetic electrons for different spreads and mean values of the angle between the incident photons and the electrons.
its average value $\bar{\gamma}_0$ by

$$\bar{\sigma}_\gamma = \frac{\int (\gamma - \bar{\gamma}_0)^2 f(\gamma) d\gamma}{\int f(\gamma) d\gamma}$$

we use four different (and possibly covering the range of typical) electron energy distribution functions:

1. Rectangular distribution with

$$f(\gamma) = \begin{cases} \frac{1}{2a} & \text{for } -a \leq \gamma - \bar{\gamma}_0 \leq +a \\ 0 & \text{elsewhere} \end{cases}$$

and $\bar{\sigma}_\gamma = a/\sqrt{3}$

2. Triangular distribution with

$$f(\gamma) = \begin{cases} \frac{1}{a^2} (\gamma - \bar{\gamma}_0 + a) & \text{for } -a \leq \gamma - \bar{\gamma}_0 \leq 0 \\ \frac{1}{a^2} (a - \gamma + \bar{\gamma}_0) & \text{for } 0 \leq \gamma - \bar{\gamma}_0 \leq a \\ 0 & \text{elsewhere} \end{cases}$$

and $\bar{\sigma}_\gamma = a/\sqrt{6}$

3. Parabolic distribution with

$$f(\gamma) = \begin{cases} \frac{3}{4a} \left[ 1 - \left( \frac{\gamma - \bar{\gamma}_0}{a} \right)^2 \right] & \text{for } -a \leq \gamma - \bar{\gamma}_0 \leq a \\ 0 & \text{elsewhere} \end{cases}$$

and $\bar{\sigma}_\gamma = a/\sqrt{6}$

4. Gaussian distribution with

$$f(\gamma) = \frac{1}{\sqrt{2\pi}a} \exp \left[ - \frac{(\gamma - \bar{\gamma}_0)^2}{2a^2} \right]$$

and $\bar{\sigma}_\gamma = a$

The spectra at $\bar{\gamma}_1 = 0$ for different standard deviations $\bar{\sigma}_\gamma$ normalized to $\gamma_0$ with $\gamma_0 = 27$ are plotted for these four distribution functions in figures 7a to d. The sensitivity of the spectrum to the used typical values of the standard deviation $\bar{\sigma}_\gamma$ is obvious. For increasing $\bar{\sigma}_\gamma$ (in all four cases) the maximum $P_{\text{max}}$ of the spectrum moves to smaller frequencies $\omega(P_{\text{max}})$, while the highest observable frequency $\omega_{\text{max}}$ goes up. (The integral $\int P(\omega_1/\omega_0) d(\omega_1/\omega_0)$
Fig. 7a and b Spectra for different standard deviations $\tilde{\sigma}_Y$ and for four typical distribution functions.
Fig. 7c and d: Spectra for different standard deviations $\tilde{\sigma}_Y$ and for four typical distribution functions.
thus is conserved). The maximum slope $\left| \frac{dP}{dw} \right|_{\text{max}}$, however, goes down with growing standard deviation $\tilde{\sigma}_\gamma$.

The frequency difference $\omega_{\text{max}} - \omega(P_{\text{max}})$, normalized to 

$$\frac{(1+\beta_0 \cos \Theta_i)(1+\beta_0)}{\delta_0^2}$$

(normalized) are plotted in figures 8 and 9, respectively, for the four different distribution functions versus the standard deviation $\tilde{\sigma}_\gamma$ (normalized to $\gamma_0$).

Although the spectra in fig.7 look relatively different for the different distribution functions, it turns out that the characteristics of the spectrum, its "width", as expressed by $\omega_{\text{max}} - \omega(P_{\text{max}})$, and its maximum slope $\left| \frac{dP}{dw} \right|_{\text{max}}$ are about equal for all four distribution functions. It is thus possible to obtain the standard deviation $\tilde{\sigma}_\gamma/\gamma_0$ of the electron energy distribution directly from the measured scattering spectrum without the necessity of knowing the type of the electron distribution function.

4. Discussion of the applicability of the described methods in typical ERA experiments

As already estimated in 2.2, the total scattered photon number for typical ruby laser and present-day (moderate) electron ring data can reach values as high as about $5 \cdot 10^5$. For negligible angular spread of the electrons as well as of the incident photons half of this number would be scattered into an angle of $\chi_0 = \arcsin \frac{1}{\gamma}$, which is about $\chi_0 \approx 2.1^0$ for $\gamma = 27$ electrons. In the experiments, however, we have a nonzero angular spread of the electrons as well as a diverging incident laser beam. The angular spread of the electrons can be estimated from the measured width of the electron ring and its focussing. In the Garching ERA device "Schuko" we measured an axial width of about $2b = 2$ cm (FWHM). With an electron number of about $N_e = 3 \cdot 10^{12}$, a magnetic field index of $n \approx 5 \cdot 10^{-3}$, a major electron ring radius of $R = 2.3$ cm and an image cylinder ("squirrel cage") radius of $R_{s.c.} = 1.6$ cm the axial focussing for unloaded electron
Fig. 8 The frequency difference for various distribution functions versus the standard deviation $\tilde{\sigma}_y/\gamma_0$.

Fig. 9 The maximum slope for various distribution functions versus the standard deviation $\tilde{\sigma}_y/\gamma_0$.
rings is given by the axial betatron oscillation tune

\[ \nu_z^2 = n + \frac{N_e r_e}{2\pi R} \left[ \frac{1}{2(R_{s.c.}/R - 1)^2} - \ln \frac{16R}{a+b} - \frac{4R^2}{8(a+b)} \right]^{1/2} \]

which numerically is

\[ \nu_z = 0.1016. \]

de is the classical electron radius.

For this estimate we used for the radial minor dimension a of the electron ring half of the axial width b.

Since the radial betatron oscillation tune \( \nu_r \) is near to \( \nu_r = 1 \), the mean radial and axial angular widths are given respectively by

\[ \Delta \phi_r = 0.435 = 25^0 \text{ (FWHM)} \]

and

\[ \Delta \phi_z = 0.087 = 5^0 \text{ (FWHM)} \]

The acceptance of the optical system of the Garching electron ring experiment, mainly limited by the hole in the magnetic field coil, however, was given by only about

\[ \Delta \phi = 0.046 = 2.6^0, \]

such that a number of only \( 10^4 \) photons enters the optical system.

In order to eliminate the stray light of the laser pulse from the optical system this fortunately can be covered by a plate of material being transparent only for the expected higher photon energies. The 12.5 \( \mu \)m thick beryllium foil, used for the experiments \[ 26 \], completely eliminates the laser stray light without reducing the flux of the scattered photons by more than 20\%, then entering a scintillator foil (type NE 102A), which moreover was covered by a thin aluminium coating in order to eliminate residual laser stray light and reflect the scintillation radiation.
If we assume - according to Meyerott et al.\textsuperscript{27} - a response of the scintillator-photomultiplier combination of about one photoelectron being produced by about three incident photons, we arrive with a current multiplication of \(10^6\) within the multiplier at a charge of about \(Q = 3 \cdot 10^{-10}\) C, which would give a voltage of as much as 2 V for a 10 nsec pulse at the usual 50 \(\Omega\) terminated cable (saturation effects of the multiplier neglected).

Thus it turns out that (in principle) the measurement of the angular distribution and also the spectrum of the scattered light should easily be possible.

In the experiment carried out\textsuperscript{26}, however, the photomultiplier could not directly be attached to the scintillator, since it was very sensitive to the high energy \(\gamma\)'s being produced by the electron beam during injection and ring forming. The photomultiplier thus had to be separated from the scintillator by a long aluminium reflector covered light tube and shielded by lead. The loss in light signal hence was reduced by a factor of about 20 (also due to several further necessary optical elements like filters).

Since it furthermore turned out that accelerated particles hit the beryllium and the scintillator foil, a collimator 10 cm in length had to be installed in front of them, taking about 80\% of the photon flux, so that we arrived at a level of about 20 mV as a rough estimate for the scattering signal.

This signal height would have been sufficient if the noise level of the multiplier, nevertheless being affected by the strong initial \(\gamma\)-burst of the electrons, would not have been of roughly the same magnitude. Unfortunately there was no time to change the experimental arrangement such that the photomultiplier would not be hit by such a strong \(\gamma\)-ray burst when being attached to the scintillator foil. A change in this direction would increase the useful multiplier signal appreciably.
Moreover a reduction of the electron ring minor radius would increase the observed signal height by roughly its fourth power, since not only the electron density goes up, but also the scattering angle decreases.

5. Conclusion

The scattering of laser light from the relativistic electrons of an electron ring can be used to determine the local angular distribution of the electrons from the angular distribution of the scattered light. The energy distribution of the electrons is obtained sensitively from the shape of the scattering spectrum. It is thus possible to measure the contribution of the closed orbit spread (due to the electron energy distribution) and the betatron oscillation width to the radial extension of the electron ring. This measurement gives the information about the importance of collective longitudinal instabilities, that cause beam blow up due to the increase in the energy spread, or of betatron resonances, which result in betatron oscillation amplitude growth, during the electron ring compression. It turns out, that the results are relatively independent of the assumed distribution functions in angle or in energy.

For moderate electron rings and for typical ruby lasers the scattered signal should be large enough for a useful measurement, although the first attempt\textsuperscript{26} of experimental application of this method at the Garching ERA device was unsuccessful due to the strong γ-ray burst of the injected electrons.
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