On some Properties of the Radiation near the Harmonics of the Electron Gyro-Frequency in Plasmas

Über einige Eigenschaften der Strahlung im Bereich der Harmonischen der Elektronen-Gyrodynamik in Plasmen

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ABSTRACT:

The position of the lines emitted by an electron in a warm plasma is calculated. From this result we deduce that the anomalously high emission in a plasma is due in some experiments to longitudinal waves, in some others to transverse waves. A mechanism for the coupling of the longitudinal (and transverse) waves with the waves which go adiabatically into vacuum is proposed. A reasonable number of suprathermal electrons can then account for the observed radiation.
1. Introduction

One of the explanations for the high emission of a plasma at the frequencies $\omega = n\omega_0 (n=1,2,...)$ or near these is based on the presence in the plasma of suprathermal electrons, which should radiate longitudinal waves. The calculations of Auer and Stone $^1$ and of Canobbio and Croci $^2$ are in agreement with the experimental data, as far as the dependence on $n$ of the amplitude of the radiated lines is concerned, and could also account for the position and width of the lines in a number of experiments (see $^1$). A point which is not yet clear is the transformation of the longitudinal waves into transverse ones, because the condition

$$\left| \frac{\omega - n\omega_0}{\omega_0} \right| \ll 1/\lambda$$

which is generally satisfied, seems to cause the coupling coefficient to be too small.

Last but not least, Landauer's experiments, the first of their kind, cannot be explained because in these the lines are centered about $\omega = n\omega_0$, while the longitudinal waves yield lines clearly shifted from the harmonics (see $^1$ and later). We believe that a theory based on the instability of electrostatic waves would be confronted with the same difficulty: it is, moreover, improbable that the lines would have the small width measured (1-2%).

Here we shall follow the method presented in $^2$ and show that in the case of the transverse extraordinary waves the radiation has the form of lines centered about $\omega = n\omega_0$ (the shift being only of the order of $\omega_0^2/\omega^2$). We also present a possible coupling mechanism of these waves and of the longitudinal ones with the transverse waves, which have an index of refraction such that $\lim_{\omega_0^2/\omega^2 \rightarrow 0} N = 1$. Comparison with the experiments shows that a reasonable number of suprathermal electrons can account for the emitted power observed.
2. Calculation of the emitted power

The power radiated by an electron per unit frequency interval in a uniform plasma with magnetic field $B_0$ is given by (see 2):

$$-\frac{dE}{d\omega} = \frac{e^2}{\pi \varepsilon_0} \text{Im} \left\{ \int_0^\infty \frac{d\omega}{\omega} \int d\mathbf{k} \sum_{m = -\infty}^{\infty} \mathbb{T}^{(m)}_{\omega \beta} \frac{D\omega}{D} \mathbb{S}(\omega - m\omega_e - k_v v_n) \right\}$$

where

$$\mathbb{T}^{(m)}_{\omega \beta} \equiv \begin{pmatrix}
\frac{m\omega_e J_m^2}{k_v^2} & \frac{i m \omega_e v_i}{J_m^2} & \frac{-i m \omega_e v_i}{J_m^2} \\
\frac{i m \omega_e v_i}{J_m^2} & \frac{v_i^2 J_m^2}{J_m^3} & \frac{-i v_i^2}{J_m^3} \\
\frac{m \omega_e v_i}{J_m^2} & \frac{i v_i^2}{J_m^3} & \frac{v_i^2}{J_m^3}
\end{pmatrix}$$

the argument of the Bessel functions is $x = k_v v_i / \omega_e$;

$$\mathbf{k} = k_v \mathbf{e}_1 + k_s \mathbf{e}_3 ; \quad B_0 = B_0 \mathbf{e}_3 ; \quad v_i \text{ and } v_L$$

are the components of the test electron velocity; the plasma electron velocity distribution is assumed to be maxwellian with thermal speed $v_T$; $\omega_e$ is the electron gyrofrequency.

$D_{\omega \beta} \equiv$ is the adjoint of

$$\mathbb{D}_{\omega \beta} \equiv \begin{pmatrix}
(k - k_v) \frac{\epsilon_{11}}{\omega^2} - \mathbb{E}_{11} & - \mathbb{E}_{12} & - \mathbb{E}_{13} \\
\mathbb{E}_{12} & k_v \frac{\epsilon_{22}}{\omega^2} - \mathbb{E}_{22} & - \mathbb{E}_{23} \\
\mathbb{E}_{13} & \mathbb{E}_{23} & (k - k_v) \frac{\epsilon_{33}}{\omega^2} - \mathbb{E}_{33}
\end{pmatrix}$$

$\mathbb{E}_{\omega \beta}(\omega, k)$ is the dielectric tensor and $D = \| \mathbb{D}_{\omega \beta} \|$.

Integration over $k_v$ reduces to the substitution (in the $\mathbb{E}_{\omega \beta}$'s):

$$\Delta = \frac{\omega - m \omega_e}{\omega} \frac{\mathbb{E}_v}{v_i} ; \quad \mathbb{S}_v \equiv \mathbb{S}_v(\omega) = \frac{\omega - m \omega_e}{\omega - m \omega_e + \frac{v_i}{v_T}} \frac{v_i}{v_T}$$
Let us consider first the case $\nu_c \ll \nu_e$. Then all the $\xi_s(m)(m = 0, \pm 1, \ldots)$ are much smaller than 1. It follows (see (3)) that all the waves with $|m| \gg L$, which are the only ones of interest here, are severely damped. When $|\nu_c| \gg \nu_e$, let us write $\omega = (n + \delta)\omega_e$ with $|\delta| < 1/2$. Then not only the term $\xi_0(n)$ is much larger than 1 (and therefore all the $\xi_s(m)$ ($m = 0, \pm 1, \ldots$)), but also all the $\xi_s(m)$ ($s = 0, \pm 1, \ldots$) for which

$$\frac{\nu_c}{\nu_e} \gg 1 + \left|\frac{m-m_0}{\delta}\right|.$$ 

It follows that the equation $D[\xi_s(m)] = 0$ has real solutions (which correspond to non-damped waves), and that the same happens for all $D[\xi_s(m)] = 0$ for which

$$1 + \left|\frac{m-m_0}{\delta}\right| \ll \frac{\nu_c}{\nu_e}.$$ 

We can therefore neglect the terms $m \neq n$ if $|\delta|$ is sufficiently small, that is, if the frequency is sufficiently near a harmonic. We shall see that the energy is mostly radiated at frequencies such that only the term $m = n$ can be important, for reasonable values of $\nu_c/\nu_e$. From now on, we shall therefore consider only this term.

When $D(\mu, \omega) = 0$ ($\mu = \frac{k^2 \nu_c^2}{c_w \omega}$) has real solutions, $\varepsilon_{1,3}$ and $\varepsilon_{2,4}$ are zero, and the same happens for $D_{1,5}$ and $D_{2,5}$.

It follows that

$$\mathcal{G}_m \equiv \left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\mu = n, \omega = \omega_e} = \frac{\varepsilon^0 \omega^0}{\nu_c^2} \sum \left[ \frac{T_{\beta \alpha}}{c_w \nu_c^4} \left(\frac{\nu_c - \nu_c}{\nu_c}\right) \frac{D_{\beta \alpha}(\mu, \omega)}{D(\mu, \omega)} \right] \delta_{\beta \alpha} \neq (4,1), (2,1)$$

where the sum is extended over the real solutions, $\mu_1$, of $D(\mu, \omega) = 0$. Averaging over the following distribution of the fast electrons
\[ f_o = \frac{\alpha}{2\pi \nu^2} e^{-\frac{\nu^2}{2\nu^2}} \delta(\nu_i - \nu_o) \] (1)

we obtain for the terms (1,1) and (2,2) (the others are analogous):

\[ \left< \delta_{n_0} \right> = \frac{\delta \nu^2 \pi \alpha}{\omega^2} \left[ \sum_{\nu} \left\{ \frac{D_{\nu}}{(\nu^2)^2} \left[ \Gamma_{\nu} \left( \frac{\nu^2}{\nu^2} \right) \right] - \mu_b \frac{\nu^2}{\nu^2} \Gamma_{\nu} \right\} \right]_{\nu = \pm \omega - \omega_e} \]

The function \( \Gamma_{\nu}(x) \equiv x^{-\nu} \Gamma_{\nu}(x) \) has the following form for \( |x| \ll 1 \):

\[ \Gamma_{\nu}(x) \approx \left( \frac{x}{\nu} \right)^{-\nu} \left( 1 + \frac{x^2}{\nu^2} \right) e^{-\nu} \]

It has a flat maximum for \( x \approx \nu^2 \), where it assumes the value \( \Gamma_{\nu} \approx 1/\nu \), then decreases as \( 1/\sqrt{x} \).

The average of the Bessel functions can, however, have a different dependence on \( \nu \), with other \( f_o \). For example, with \( \hat{f}_o \propto \delta(\nu_i - \nu_o) \) the first term goes as \( \nu^{-2/3} \), and with a more realistic \( f_o \) it would probably behave in a yet more favourable way.

The factor \( D'(\nu, \omega) \) can be zero, in the limit of no absorption. Then the group velocity

\[ \nu_g = -\frac{\nu^2}{\nu_o} \nu_o \frac{\partial}{\partial \omega} \left( \frac{\nu^2}{\nu_o} \frac{\partial}{\partial \omega} \right) \]

is also zero.

Taking account of the small imaginary part of \( R_{\nu} \)

\[ \delta \equiv \nu_{\nu} - \nu_{\nu^2} = \frac{1}{|k_o|} \]

(2)
which would be responsible for negligible Landau damping, we can write for the terms $D_n$ and $D_{2n}$

$$\left<\xi_n\right> = \frac{e^{-\nu/c}}{v_n} \sum \frac{D_j}{1 + D_j} \frac{1}{1 + D_j} \left\{ \frac{D_{2n}}{1 + D_{2n}} \right\}$$

Before we calculate $\left| D_{\ldots} \right|$, we shall determine the frequencies and wavelengths where $\nu_\parallel = 0$.

We remember that, under the condition $|\eta| \ll \frac{\omega}{\omega_0}$,

$$\eta = 1 + \frac{\nu}{\nu_0} \left( 1 + \pi \int_0^\infty \cos \omega t \left( \frac{\omega}{\omega_0} \right)^2 \right)$$

$$\eta \approx \begin{cases} 1 - \frac{\nu}{\nu_0} & (|\eta| \ll 1, \text{ and } \nu \gg 1 \text{ for the third term}) \\ 1 + \frac{1}{\nu_0^2} (1 - \frac{\nu}{\nu_0}) & (|\eta| \gg \nu_0 \text{ or } |\eta| \gg 1) \end{cases}$$

the dispersion relation separates into the following three equations

$$\begin{align}
\frac{b}{D_0} = \frac{\nu_0}{\nu_0} \left( \nu_0 - \eta \right) + \frac{\nu}{\nu_0} \\
\frac{b}{D_0} = -\frac{\nu_0}{\nu_0} \frac{D_0}{D_{2n}} \\
\frac{b}{D_0} = -\frac{\nu_0}{\nu_0} \frac{D_0}{D_{2n}}
\end{align}$$

where

$$b = -\frac{\pi}{\omega_0 \pi \nu}$$

$$D_0 = \frac{(\eta - 1)\nu}{\nu_0} + \frac{\nu}{\nu_0} \left( \nu_0^2 - \eta + 2 \frac{\nu_0}{\nu_0} (\eta - 1) \right)$$

$$D_1 = \frac{2 (\eta - 1) \beta + \frac{\nu}{\nu_0} \left( \nu_0^2 - 2 \eta + 2 \frac{\nu_0}{\nu_0} (\eta - 1) \right) + \eta \beta}{\nu_0}$$
\[ D_x = \nu \left( \beta^2 + 2 \beta - \frac{\nu}{\kappa^2} \right) \]

\[ \beta = \frac{\Gamma'_x}{\Gamma_x}. \]

(see 3).

The first equation corresponds to transverse ordinary waves.

Then \( \nu \) can be zero for \( \lambda = \frac{r_n}{n} \) and \( \frac{|\nu - n|}{\nu} \approx \nu \frac{\nu_c}{\kappa^2} \frac{\omega_p^2}{\omega_p^2} \).

However, with the small imaginary part (2), \( \left| \frac{D_{10x}}{D_{10}} \right| \) is smaller than \( \left| \frac{D_{12x}}{D_{12}} \right| \) and \( \left| \frac{D_{14x}}{D_{14}} \right| \) (calculated for the transverse extraordinary and the longitudinal waves, respectively) by a factor \( \nu \frac{\nu_c}{\kappa^2} \) and can therefore be neglected. The second equation corresponds to quasi-transverse waves:

\[ \left| \frac{E_y}{E_x} \right| = \left| \frac{D_{10x}}{D_{10}} \right| = \left| - \frac{E_n}{E_n} \right| \gg 1 \]

\[ \text{for } \lambda = \frac{r_n}{n}, \quad \frac{i E_x}{E_y} \approx \frac{\nu - n}{\nu} \frac{\nu_c}{\kappa^2}. \]  

The condition \( \nu = 0 \) is satisfied for

\[ \lambda = \frac{r_n}{n} \quad \text{and} \quad \frac{|\nu - n|}{\nu} \approx \nu \frac{\nu_c}{\kappa^2} \frac{\omega_p^2}{\omega_p^2}. \]  

The third equation corresponds to quasi-longitudinal oscillations because

\[ \left| \frac{E_x}{E_y} \right| = \left| \frac{D_{14x}}{D_{14}} \right| \gg 1 \]

\[ \text{for } \lambda = \frac{r_n}{n}, \quad \frac{i E_x}{E_y} = \nu \frac{\nu_c}{\kappa^2}. \]
Using the approximations given in \(3\) for \(\beta = \frac{v}{c} = \frac{v}{c}\), one finds for the waves \(4\) a \(\lambda\) somewhat smaller, but of the order of \(\frac{r_n}{n}\) and

\[
\frac{|\tau_{\omega r} \pi r|}{\pi v} \approx \frac{d}{\nu|\eta|}.
\]

The maximum of the radiation is therefore clearly shifted from the harmonics.

A particularly interesting case is that of the quasi-longitudinal waves in the neighbourhood of the hybrid frequency \(\omega = \omega_{\rho} + \omega_{\nu} \). When \(\eta\) is so small that \(|\eta|^{1/2} \ll 1\) it follows from eq. \(3\) that

\[
\lambda \approx \frac{r_n}{n} \quad ; \quad \frac{|\tau_{\omega r} \pi r|}{\pi v} \gg 1
\]

in agreement with \(7\). The lines are then centered about

\[\omega = (m + \nu) \omega_{\nu} .\]

In order to determine now \(|D_{\omega r}\)| we calculate the modification of the solutions \(\tau_{\omega r}\) of \(D = 0\), due to the presence of the small imaginary part \(\delta\).

From \(3\) and \(4\) (we do not consider the transverse ordinary waves) it follows that the modified solutions are approximately

\[
\tau_{\omega r} + i \eta \frac{\omega}{\nu} \delta \tau_{\omega r} = \left(1 + i \frac{\omega}{v_0 c} \delta\right) \tau_{\omega r}
\]

for both transverse extraordinary and longitudinal waves.

Developing now in a Taylor series the function \(D'(\tau_{\omega r} (1 + i \eta \delta))\) we obtain

\[
|D_{\omega r}'| \approx \frac{\eta |\delta|}{\nu v} \frac{\sqrt{\delta}}{v_0}
\]

for the longitudinal waves

\[
|D_{\omega r}'| \approx \frac{\eta |\delta|}{\nu v} \frac{\sqrt{\delta}}{v_0}
\]

for the transverse extraordinary waves.

In a non-uniform plasma these formulae are approximately valid as long as in their Taylor expansion at the density
which satisfies (7) or (8), the first term is the dominant one. In this way it can be shown that (9) are approximately valid in the interval

$$\tilde{\omega}_p^2 < \frac{\omega}{c} < \tilde{\omega}_p^2 + \Delta \omega_p^2$$

where

$$\Delta \omega_p^2 = \delta \omega_p^2.$$ (10)

The energy emitted per unit time per unit frequency by electrons uniformly distributed with density \( \alpha \) in a region of width \( l \) is: (one can, of course, neglect the small variation of the density over a Larmor radius)

$$\frac{\omega}{c} \frac{\Omega}{\gamma} \Gamma < \frac{\delta}{\gamma} \Gamma < \frac{\delta \Omega}{\gamma} \Gamma$$.

Choosing for \( l \) the length defined by (10), we obtain

$$\frac{\omega}{c} \frac{\Delta \omega_p^2}{\gamma \omega_p} \Gamma < \frac{\delta \Omega}{\gamma} \Gamma < \frac{\delta \Omega}{\gamma} \Gamma \equiv E_n.$$ (10)

The imaginary part of \( D \) appears now in the numerator. From eqs. (5) and (7) it follows that the most important term for the transverse waves is that proportional to \( D_{2r} \); for the longitudinal waves it is the one proportional to \( D_{2r}. \)

We can therefore write

$$E_n, t = \frac{e^2 \omega}{\omega_p^2} \alpha \frac{\omega_p^2}{\gamma \omega_p \omega_p} \frac{c^2}{\gamma^2} \frac{\nu^2}{\nu^2} \frac{r^2}{\nu^2} \frac{r^2}{\nu^2}$$

$$E_n, r = \frac{e^2 \omega}{\omega_p^2} \alpha \frac{\omega_p^2}{\gamma \omega_p \omega_p} \nu \frac{\nu^2}{\nu^2} \frac{r^2}{\nu^2}.$$

\( E_n, t \) is larger than \( E_n, r \) by a factor \( c^2/\omega_p^2 \).

However, it will be shown that only a fraction \( \nu^2/\omega_p^2 \) of \( E_n, t \) gives rise to a Poynting vector, the rest corresponding to polarization losses. In fact, from the Maxwell equations it follows for the electric displacement

$$D_r = -4\pi i \frac{c}{F} \left\{ \epsilon_0 \frac{D}{\nu^2} \frac{d}{d\nu} \right\}.$$
where $F^{-1} \{ \cdot \}$ indicate the inverse Fourier transform, and

$$\tilde{f} = e^{-i \frac{\pi}{2}} F^{-1} \{ \overline{v}(x, t) \}.$$  

Taking for simplicity $v_n = 0$, at the time where $\overline{v} \equiv v_x$, we obtain

$$\frac{\partial}{\partial t} \approx -4 \pi e^{-i \frac{\pi}{2}} \frac{\partial}{\partial \omega} \frac{f_x}{\omega} \right]$$

that is,

$$\frac{2 \frac{\partial}{\partial t}}{\gamma e} \approx -4 \pi e^{-i \frac{\pi}{2}} v_x,$$  

where the terms neglected are of the order

$$\left| \frac{D_{\perp}}{D_{\parallel}} \right| \approx \frac{\nu e^{-i \frac{\pi}{2}}}{\nu c} \quad \text{(from (6))}.$$  

From the fact that the energy emitted is $-e \overline{v} \cdot \overline{E}$, it follows that the divergence of the Poynting vector is of the order $\frac{\nu e^{-i \frac{\pi}{2}}}{\nu c} < \langle \mathcal{E} \rangle$ and that the largest part of the emitted energy is due to polarization losses.

The ratio of the Poynting vectors connected with $\langle \mathcal{E} \rangle$ and $\langle \mathcal{E} \rangle$ is therefore

$$\frac{S_{\perp}}{S_{\parallel}} \approx \nu.$$  

With $\omega = 6.14 \times 3 \times 10^{-5} \text{ s}^{-1}$, $v_n = 3 \times 10^7 \text{ cm s}^{-1}$,

we obtain:

$$E_{\nu, \tau} \approx 1.5 \times 10^{-17} \frac{n}{d} \frac{\nu e^{-i \frac{\pi}{2}}}{\gamma c \nu_d} \left[ \text{erg cm}^{-2} \right].$$  

3. The transmission coefficient

We shall develop here an idea touched on in 3).

The only wave which has an index of refraction such that

$$\frac{\beta_n}{\nu_n} \approx 1$$

$$\nu_n < 0$$
is the transverse "macroscopic" one, with
\[ N_\perp \equiv M_\perp = \eta - \frac{(\gamma - 1) \nu}{\nu} \cdot \]
(for \( \gamma \) one has to take the limit \( |\nu| \ll \nu \)).

\( N_\perp \) is real if
\[ \omega_\perp < \omega - \nu (\omega - \omega_x) \tag{11} \]
and in the neighbourhood of the hybrid frequency, namely for
\[ \omega (\omega + \omega_x) > \omega_\perp > \omega^2 - \omega_x^2 . \tag{12} \]
The longitudinal and transverse waves (3-4) have a real index of refraction only for densities larger than a critical density where \( \nu_f = 0 \), as eqs. (3) and (4) show.

Because the emitted radiation has a wavelength \( \lambda \approx \frac{\nu_f}{n} \) and the density gradients in a plasma are generally such that
\[ \left| \frac{\text{grad} \; \nu_f}{\nu_\perp} \right| \ll \frac{1}{\nu_f^n} , \]
we are usually allowed to use the WKB method. The emitted waves will therefore propagate back and forth in a plasma column defined by the density corresponding to \( \nu_f = 0 \) for the considered frequency. In the neighbourhood of \( \nu_f = 0 \) however, the WKB method is no longer valid, because then
\[ \frac{d k}{d x} = - \frac{\omega_\perp}{2 \nu_\perp^2} - \frac{\nu_\perp}{\nu_f} \approx \frac{1}{\nu_f} \]
becomes infinite. At these frequencies (where at the same time the radiation reaches its maximum) we can expect good coupling with the other waves.

Because in the neighbourhood of the density where \( \nu_f = 0 \) the equation \( \left| \frac{\text{grad} \; \nu_f}{\nu_f} \right| \lambda \gg 1 \) is satisfied, we can consider this region as infinitely thin and treat it as a surface separating two mediums with different indices of refraction.
The only waves which can be coupled by this mechanism with the "macroscopic" one are those with a frequency such that \( \omega_f \) is zero for densities in the interval (11) (not in (12), because the condition \( k \lambda_0 < 1 \) for \( \lambda = \frac{c}{\omega} \) corresponds to \( \omega_f \omega_f \). Note also that in the interval of \( \omega_c \) about \( \omega_c \), where \( |\eta' \psi'| \ll 1 \), the "macroscopic" wave goes into a quasi-longitudinal one (see 3).

That is, those with

\[
|\nu - \nu_c| \ll \frac{\nu_c}{c}
\]

for the transverse waves, and

\[
|\nu - \nu_n| \ll \frac{\nu_n}{c}
\]

for the longitudinal waves.

Let us now take the plane \( x = 0 \) as the surface where \( \sigma_j = 0 \).

Then if we assume that for \( x < 0 \) there is only an incident and a reflected quasi-longitudinal (or transverse extraordinary) wave, and for \( x > 0 \) a "macroscopic" transverse wave, it follows from the Maxwell equations that the y-component of the "macroscopic" electric field in the second medium is approximately equal to the y-component of the incident electric field, because \( \eta_\perp \) is much smaller than the indices of refraction which follow from (7) and (8).

For the contribution to the Poynting vector in the second medium from the quasi-longitudinal and the transverse waves respectively, in the case where \( f_0 \) is given by (1), we can then write

\[
S_{\perp, \perp} = e^{i \omega_f} \frac{\partial \omega_\perp}{\partial \partial_\perp} \frac{\nu_\perp}{c} \left( \frac{\omega_f}{\nu_\perp} \right) \frac{\eta_\perp \nu_\perp}{\nu_\perp}
\]

\[
S_{\perp, \tau} = e^{i \omega_f} \frac{\partial \omega_\perp}{\partial \partial_\perp} \frac{\nu_\perp}{c} \left( \frac{\omega_f}{\nu_\perp} \right) \frac{\eta_\perp \nu_\perp}{\nu_\perp}
\]
The passage of the energy through the region where \( \kappa_\tau > 0 \) (that is, where \( \omega_0 < \omega_\tau < \omega_\tau + \omega_e \) ) should not cause difficulties, since the dependence of \( \Delta \) on \( \omega_\tau \) is such that the absorption in this region should be negligible. A density of the fast electrons of the order \( \delta \omega_\tau^2 \omega_e^{-3} \) is enough to explain the observed power.

We conclude that experiments like those of Landauer, where the lines are very thin and centered about \( \omega = \omega_0 + \omega_e \) cannot be explained in terms of longitudinal waves. We believe that they cannot be explained either in terms of a possible theory of electrostatic instability, for the same reason. The radiation due to transverse waves, on the contrary, seems to correspond to Landauer's experiments.

It is probable that in \(^4\) both kinds of waves have been observed.

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Bibliography


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