Dynamo and Alfvén effect in MHD turbulence

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This extended abstract reports a spectral relation between residual and total energy, \( E^R_k = |E^M_k - E^K_k| \) and \( E_k = E^K_k + E^M_k \) respectively, as well as the influence of an imposed mean magnetic field on the spectra. The proposed physical picture, which is confirmed by accompanying direct numerical simulations, embraces two-dimensional MHD turbulence, globally isotropic three-dimensional systems as well as turbulence permeated by a strong mean magnetic field. The results have direct implications on the current understanding of the energy cascade in MHD turbulence.

In the following reference is made to two high-resolution pseudospectral direct numerical simulations of incompressible MHD turbulence which we regard as paradigms for isotropic (I) and anisotropic (II) MHD turbulence. The dimensionless MHD equations

\[
\begin{align*}
\partial_t \omega &= \nabla \times \left[ \mathbf{v} \times \omega - \mathbf{b} \times (\nabla \times \mathbf{b}) \right] + \mu \Delta \omega \\
\partial_t \mathbf{b} &= \nabla \times (\mathbf{v} \times \mathbf{b}) + \eta \Delta \mathbf{b} \\
\nabla \cdot \mathbf{v} &= \nabla \cdot \mathbf{b} = 0.
\end{align*}
\]

are solved in a 2\( \pi \)-periodic cube with spherical mode truncation to reduce numerical aliasing errors [1]. The equations include the flow vorticity, \( \omega = \nabla \times \mathbf{v} \), the magnetic field expressed in Alfvén speed units, \( \mathbf{b} \), as well as dimensionless viscosity, \( \mu \), and resistivity, \( \eta \).

![Figure 1: Total (solid), kinetic (dashed), and magnetic (dotted) energy in a 1024³ simulation of decaying isotropic MHD turbulence (left) and in a 1024² × 256 simulation of anisotropic turbulence permeated by a strong mean magnetic field, \( b_0 = 5 \) (right, spectra are based on field perpendicular fluctuations). The dash-dotted line in the graph on the left illustrates a \( k^{-3/2} \) power-law while the dashed horizontals indicate \( k^{-5/3} \)-behavior (left) and \( k^{-3/2} \)-scaling (right). The dash-dotted curve on the right shows the high-k part of the field-parallel total energy spectrum. The inset displays the difference in the perpendicular total energy spectrum when switching resolution from 512² (dash-dotted) to 1024² (solid).](image)

Simulation I evolves globally isotropic freely decaying turbulence represented by 1024³ Fourier modes. Total kinetic and magnetic energy are initially equal with \( E^K = E^M = 0.5 \). The dissipation
Figure 2: Compensated and space-angle-integrated residual energy spectrum, $E^R_k = |E^M_k - E^K_k|$, for the same systems as in Fig. 1 (isotropic:left, mean magnetic field: right). The dash-dotted line depicts scaling expected for a total energy spectrum following Iroshnikov-Kraichnan scaling.

parameters are set to $\mu = \eta = 1 \times 10^{-4}$. Case II is a $1024^2 \times 256$ forced turbulence simulation with an imposed constant mean magnetic field of strength $b_0 = 5$ in units of the large-scale rms magnetic field $\sim 1$ with $\mu = \eta = 9 \times 10^{-5}$.

Fourier-space-angle integrated spectra of total, magnetic, and kinetic energy for case I are shown in Fig. 1 (left). To neutralize secular changes as a consequence of turbulence decay, amplitude normalization assuming a Kolmogorov total energy spectrum, $E_k \rightarrow E_k/(\varepsilon \mu^3)$, $\varepsilon = -\partial_t E$, with wavenumbers given in inverse multiples of the associated dissipation length, $\ell_D \sim (\mu^3/\varepsilon)^{1/4}$. Clearly, Kolmogorov scaling applies for the total energy in the well-developed inertial range, $0.01 < k < 0.1$.

In case II, pictured in Fig. 1 (right), strong anisotropy is generated due to turbulence depletion along the mean magnetic field, $b_0$. This is visible when comparing the normalized and time-averaged field-perpendicular one-dimensional spectrum, $E_{k_\perp} = \int \int dk_1 dk_2 E(k_\perp, k_1, k_2)$ (solid line) with the field-parallel spectrum, defined correspondingly and adumbrated by the dash-dotted line in Fig. 1 (right).

While there is no discernible inertial range in the parallel spectrum, its perpendicular counterpart exhibits an interval with Iroshnikov-Kraichnan scaling, $E_{k_\perp} \sim k_\perp^{-3/2}$ [2, 3]. This is in contradiction with the anisotropic cascade phenomenology of Goldreich and Sridhar for strong turbulence predicting $E_{k_\perp} \sim k_\perp^{-5/3}$ [4].

The observation that field-parallel fluctuations are restricted to large scales while the perpendicular spectrum extends more than half a decade further suggests that the strong $b_0$ constrains turbulence to quasi-two-dimensional field-perpendicular planes as is well known and has been shown for this particular system [5].

Another intriguing feature of system II is that $E^K_k \approx E^M_k$ with only slight dominance of $E^M$ (cf. Fig. 1, right) in contrast to the growing excess of spectral magnetic energy with increasing spatial scale for case I. Both states presumably represent equilibria between two competing nonlinear processes: field-line deformation by turbulent motions on the spectrally local time scale $\tau_{NL} \sim \ell/\nu_L \sim (k^3 E^K_k)^{-1/2}$ leading to magnetic field amplification (turbulent small-scale dynamo) and energy equipartition by shear Alfvén waves with the characteristic time $\tau_A \sim \ell/b_0 \sim (k b_0)^{-1}$ (Alfvén effect).

By using the spectral EDQNM equation for the residual energy in spectrally local and non-local approximations [6] and by assuming that the residual energy is a result of a dynamic equilibrium between turbulent dynamo and Alfvén effect, one obtains for stationary conditions and in the inertial
with $\tau_A \sim (kb_0)^{-1}$, where $b_0$ is the mean magnetic field carried by the largest eddies, $b_0 \sim (E^M)^{1/2}$, and by re-defining $\tau_{NL} \sim \ell/(v^2 + b^2_{\parallel})^{1/2} \sim (k^3 E_k)^{-1/2}$. The modification of $\tau_{NL}$ is motivated by the fact that turbulent magnetic fields are generally not force-free so that magnetic pressure and tension contribute to eddy deformation as well.

Apart from giving a prediction which allows to verify the proposed model of nonlinear interplay between kinetic and magnetic energy, relation (4) also has some practical utility. It is a straightforward consequence of (4) that the difference between possible spectral scaling exponents, which is typically small and hard to measure reliably, is enlarged by a factor of two in $E^R_k$. Even with the limited Reynolds numbers in today’s simulations such a magnified difference is clearly observable (e.g. dash-dotted lines in Figs. 1 and 2).

In summary, based on the structure of the EDQNM closure equations for incompressible MHD a model of the nonlinear spectral interplay between kinetic and magnetic energy is formulated. The quasi-equilibrium of turbulent small-scale dynamo and Alfvén effect leads to a relation linking total and residual energy spectra, in particular $E^R_k \sim k^{-7/3}$ for $E_k \sim k^{-5/3}$ and $E^R_k \sim k^{-2}$ for $E_k \sim k^{-3/2}$. Both predictions are confirmed by high-resolution direct numerical simulations of isotropic turbulence exhibiting Kolmogorov scaling and forced anisotropic turbulence displaying Iroshnikov-Kraichnan scaling perpendicular to the mean field direction. The findings limit the possible validity of the Goldreich-Sridhar phenomenology to MHD turbulence with weak mean magnetic fields and emphasize the important role of the Iroshnikov-Kraichnan picture for a large class of turbulent MHD systems.

References