Assessment of Global Stellarator Confinement:

Status of the International Stellarator Confinement Data Base


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Status of the International Stellarator Confinement Data Base(a)

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Abstract

Further exploration of confinement trends in the evolving International Stellarator Confinement Database is reported. The impact of configurations on confinement is confirmed and the performance close to operational limits discussed. Model comparison techniques allow for tests of physical models against data. Correlations of configuration-describing parameters against an empirical confinement enhancement factor are investigated.
1. Introduction

Because of their three-dimensional nature, stellarator/heliotron devices cover a large magnetic configuration space. Since the ultimate goal of stellarator research is to develop an alternative fusion reactor concept, the exploration of the most promising configurations requires comparative assessments of the plasma performance and how different aspects of a 3D configuration influence it.

To this end, the International Stellarator Confinement Database (ISCDB) project was been re-initiated in 2004 to extend the ISS95 [1] database to roughly 3300 discharges from nine different devices. The revision of a data set restricted to comparable scenarios led to the ISS04 scaling law [2] which confirmed the general tendencies of ISS95, but also revealed clearly the necessity of incorporating configuration-descriptive parameters. In other words, an extension beyond the set of regression parameters used for ISS95/ISS04 appears to be necessary, and candidates, such as the flux-surface elongation are investigated. Since grouping of data is a key-issue for deriving ISS04, basic assumptions are revisited in this paper, e.g. the dependence on the heating scheme.

Furthermore, the statistical approaches used are assessed with respect to their impact on the scaling. A crucial issue is the weighting of data groups which is discussed in terms of error-in-variable techniques. Bayesian model comparison is employed for testing scaling hypotheses based on scaling invariance principles thus allowing the assessment of applicability of theory-based scaling laws on stellarator confinement. Again, subsets of ISCDB are used in order to ensure comparability of the data under investigation.
2. The ISCDB dataset and the ISS04 stellarator scaling

Plotting the data from the new ISCDB data set against the original ISS95 scaling relation (Fig. 1) shows subgroup clusters around an overall trend. Systematic offsets are visible between the various groups of data: for example, the LHD data indicate enhanced performance compared to the ISS95 prediction. This motivates to confirm specifically the gross scaling of regression parameters (minor and major radius \((a, R)\), mean density \(n (10^{19} m^{-3})\), absorbed power \(P (MW)\), magnetic field \(B (T)\) and rotational transform \(\iota\)).

The tokamak data [3] in Fig. 1 are shown for comparison. This comparison is qualified because of differences in the set of regression parameters and assumptions on the rotational transform profiles [1] as follows. From the tokamak database the elongation corrected minor radius \((a_{tok} \sqrt{\kappa_{tok}})\) and \(\iota_{2/3} = 1/q_{2/3}\) was calculated using an \(\iota\)-profile as \(\iota(\rho) = \iota_a \times \left(1 - (1 - \rho^2)^{\frac{3}{2}}\right)\rho^{-2}\) where \(\rho\) is the effective radius. \(\iota_a (\rho = 1)\) was derived from \(I_p = 2.5 a^2 B_t R^{\frac{3}{2}} (1 + \kappa^2) \iota_a\) which relates the plasma current \(I_p\) to the edge rotational transform and the toroidal magnetic field \(B_t\). The ITER prediction is consistent with ELMy-H mode data. Although a number of non-validated assumptions enter the comparison, the stellarator data join the high confinement data from tokamaks in the range of confinement times covered. Hence, there is no indication of a degraded confinement – in particular from the data of the largest device, LHD.

A second conclusion can also be drawn from Fig. 1: Data from different devices form clusters whose coarse tendency may differ from the general trend of ISS95. A closer inspection even shows subgroups within discharges of a single device. This observation led to the \textit{ad hoc} assumption of ISS04: stellarator configurations are
different, but a common scaling in the leading regression parameters is to be investigated. Technically, ISS04 assigns to each subgroup a configuration parameter which is determined as the ratio of experimentally determined confinement times and a reference scaling (here ISS95 was chosen, $f$ is normalized to the W7-AS (limiter) subgroup) [2].

The regression of renormalized data results in:

$$\tau_E^{ISS04} = 0.134 a^{2.28} R^{0.64} P^{-0.61} n_e^{-0.54} B^{0.84} \tau_{2/3}^{0.41}$$

where $a^*$ reflects the minor radius dependencies not contained in the dimensionless scaling parameters (normalized collisionality $\nu^*$, normalized gyro-radius $\rho^*$ and plasma beta $\beta$). A vanishing $a^*$ dependence indicates the scaling to be dimensionally correct.

The RMSE value of the regression is 0.0267 if the weighted renormalized confinement time is employed as regression data. Using the original confinement time data gives an RMSE of 0.2366.

The scaling law ISS04 confirms the findings of ISS95 with respect to the scaling exponents which reads:

$$\tau_E^{ISS95} = 0.079 a^{2.21} R^{0.65} P^{-0.59} n_e^{0.51} B^{0.53} \tau_{2/3}^{0.4}$$

More details, in particular on assessments of assumptions on the regression ansatz can be found in Ref. [2]. Figure 2 shows the ISS04 scaling results in conjunction with the configuration factors used for deriving the ISS04 scaling law. In addition to the
ISS04 subset of ISCDB (1721 from 3335 datasets) the remaining ISCDB data are plotted in Fig. 2 for the purpose of comparison. The configuration factor was derived as for the ISS04 subgroups.

The dependence of $f$ on the reference scaling is vanishing if the derived scaling is used iteratively. In addition, the normalization factor ansatz is independent from the choice of reference; here ISS95 [1], Lackner-Gottardi [5] and LHD [6] scaling laws were employed as initial reference. For all references the iterative determination converges within one standard deviation of the regression exponents results against

$$\tau_{E}^{ISS04,\infty} = f(\text{configuration}) \times 0.137 a^{2.40} R^{0.64} P^{-0.57} n_{e}^{0.55} B^{0.90} \rho_{2/3}^{0.09}$$

The RMSE value of the scaling is 0.0262 (0.2361 if the original data were used as reference). This scaling law is close to ideal gyro-Bohm scaling laws [7]. The dependence on the rotational transform differs substantially from ISS95, but nevertheless the $\tau$ dependence is significant regardless which reference is initially assumed.

Due to technical settings and physical constraints the conditioning of a global confinement data-set from fusion devices is always restricted. This appears, e.g. in Ref. [2] as an apparent correlation of the scaling assumptions (weighting, normalization) and the scaling exponents. Such correlations were described in detail in Ref. [2] and the strategy to identify the role of correlations and data clustering is a detailed assessment of the data-sets both with respect to the physical properties (next section) as well as consistency of scaling models with the data as discussed in the over-next section. The collinearity between $f$ and other engineering parameters is to be studied in future.
3. Assessment of ISCDB data

The ISS04 scaling was derived from a subset of the International Stellarator Confinement Database. Partly, the subset for scaling was restricted to data which are expected to store most of their energy as thermal energy. Therefore, density and power scans from the HSX device were not included in scaling studies.

The dependence on the heating scheme was investigated by applying the ISS04 scaling procedure on electron cyclotron heated plasma data. The resulting scaling exponents are consistent with the ISS04 scaling law except for the scaling in density and magnetic field \( \tau \sim n^{0.63\pm0.02}B^{0.63\pm0.03} \) instead of \( \tau \sim n^{0.54\pm0.01}B^{0.84\pm0.02} \). The differences, however, may in part be attributed to a density – magnetic field correlation in the data of ECRH plasmas (correlation coefficients \( r_{ECH} = 0.81 \) between log(n) and log(B) vs \( r_{ISS04} = 0.18 \) for the full data set).

An example of such a subset is a density scan of the confinement time in the transition from normal confinement to the high density H-mode (HDH) in Wendelstein 7-AS. This transition is shown in the left frame of Fig. 3. This example shows that as the density rises, the energy confinement time jumps to a different confinement mode. At still higher densities, confinement then degrades with density as the plasma becomes detached from the divertor targets in divertor operation [8,9].

A second example of regression parameter variations is the comparison of the \( \chi \) dependence of a W7-AS subset [10] and TJ-II data [11] shown in the right plot of Fig. 3. Taking account for the configuration factor, the \( \chi \) scaling of different configurations can be compared. This approach allows one to make use of the large parameter variations of an inter-machine database. Since ISS04 corresponds to the \( \tau^{\text{exp}} = f \tau^{\text{ISS04}} \)
line in the right plot of Fig. 3 the gross scaling of $\lambda$ appears to be consistent from the inter-machine comparison. However, the variation of the data indicates possible uncertainties even in the gross scaling; for comparison the green line indicates what would be expected for vanishing $\lambda$ dependence in ISS04. The fine structure of the $\lambda$ dependence due to low order rationals is a salient feature of low-shear stellarator confinement and is extensively documented and discussed in [10,11].

Beyond scaling behavior the documentation of high confinement modes is also started to be addressed in ISCDB. Fig. 4 shows selected examples of the configuration factor under mode transitions, e.g. for HDH bifurcations and a single example for a L-to H-mode transition in W7-AS. Please note that the renormalization factors of the single L-H-mode transition are within the error margins of the renormalization factor of W7-AS limiter discharges (see Fig. 2) but show the confinement enhancement in H-modes. The configuration scan in LHD documents the enhancement of confinement due to inward shift of the magnetic configuration [12,13].

4. Assessment of physical models

A crucial issue in the assessment of scaling laws is the estimate of trustworthiness not only for reference purposes but ultimately for a predictive statement. One possibility to link the scaling relation with physics models is to derive dimensional constraints on the scaling exponents from invariance requirements of the basic equations describing plasma behavior connected with collisionality $v^*$ or plasma $\beta$[14].

In order to compare these physics models on the basis of data from the ISCDB, Bayesian probability theory is employed. For model comparison an inclusion of uncertainties is necessary. Neglecting these uncertainties means assigning equal weight
to every entry in the database. However, the uncertainties of the data vary substantially being the result of different diagnostics measured in different machines during different experimental campaigns.

The error statistics taking into consideration the uncertainty both in the regression target \( \tau_E \) with error \( \sigma_\tau \) and the machine variables (so called errors in variables approach), can be written in terms of a probability distribution, i.e. likelihood, as [15,16]:

\[
p(\bar{x}, \tau_E | \bar{\alpha}, \bar{\sigma}) = \frac{\text{const}}{(2\pi)^{K/2} \prod_k \sigma_k} \exp \left[ -\frac{1}{2} \left( \frac{\tau_E - \sum_k \alpha_k x_k}{\sigma_\tau} \right)^2 \right] 
\]

(4)

where \( \bar{x} \) represents the \( K = 6 \) ISS regression variables \( (a,R,P,n,B,\iota) \). \( \alpha_k \) and \( \sigma_k \) are their respective regression exponents and errors. The denominator in the argument of the exponent marks the difference with ordinary least squares fitting.

Since the present content of ISCDB covers different ranges of collisionality and plasma \( \beta \), the validity of low/high collisionality and low/high beta models can be performed. The model probability is proportional to the posterior probability of the data, i.e. marginalizing the above likelihood over all \( \alpha_k \) and normalizing the result with respect to the available models. The sum of the model probabilities for the four models becomes unity. A methodological summary is to be found in Ref. [17,18].

A first study focuses on the low/high \( \beta \) and \( \nu^* \) data from W7-AS. As can be seen in the right graph of Fig. 5, the low \( \beta \) data set is given by \( \nu^* < 23 \) and \( \beta < 0.004 \) giving a
total of 223 shots, on the other hand the high $\beta$ data set to $\nu > 100$ and $\beta > 0.015$ giving a total of 43 shots. As for ISS04, the rotational transform subdivides the W7-AS data and shots with $\iota < 0.4$ were taken into account only.

The result of model comparison calculations is shown in the left graph of Fig. 5. The models under consideration are reviewed in Ref. [19]. It appears that the set designated as low-$\beta$ data can be described by a collisional low $\beta$ Connor-Taylor model (electrostatic Fokker-Planck). It is to be noted that this subset result is not in agreement with the dimensionless formulation of ISS04. Since the model probabilities do not prefer a distinct solution, the model assessment fails for the high $\beta$ data. The methods applied now allows one for a successive enhancement of significance of model assessment and is used as a guideline for future work, particularly for the $\beta$ scaling of confinement in stellarator/heliotron devices.

5. Configuration factor assessment

ISS04 is derived with renormalized data and it confirms the previous scaling law results. The variety of configuration factors derived, however, also allows one to explore correlations of the renormalization factor with configurations. The configuration factor should represent different physics/configurations in subgroups. Furthermore, a practical demand for a configuration factor is its accessibility.

The LHD configuration scan data clearly shows the importance of the magnetic configuration for confinement (cf. Fig. 4). Therefore, in a first step, properties of the magnetic field structure are explored with respect to the derived configuration factor.
Tokamak confinement scaling laws indicate significant dependence on the elongation [20]. Moreover, the ITER design [21] also includes the trianularity, indicating again the relevance of magnetic geometry for confinement.

The purely geometrical definition of elongation used in tokamaks fails for stellarators being insufficient to characterize drift orbits in the helical field. Instead, a definition employing magnetic (Boozer) coordinates is used with generalizes the elongation in 3-d geometries (see [22]):

$$\kappa = \left( \frac{e_t}{b_{10}} \right)^2$$  \hspace{1cm} (5)

where $b_{10}$ describes the average toroidal curvature in these coordinates. This definition is equivalent to the geometrical definition in the limiting 2D (tokamak) case.

A number of caveats must be raised if this factor is used as a configuration factor for stellarators:

- The factor $b_{10}$ increases linearly with minor radius in classical stellarator configurations and this dependence should cancel the minor radius factor in the inverse aspect ratio $\varepsilon_t = r/R$. However, there may be a substantial deviation from linear behavior near the last closed flux surface, for example, in the case of shifted heliotron configurations. Therefore, we chose the $\kappa$ value at 2/3 of the minor radius just as we choose for ISS95 and ISS04 to evaluate $\varepsilon$ at the same radius.
- The use of elongation must fail for quasi-helically symmetric configurations since the corresponding \( b_{10} \) component vanishes in these configurations.

At high beta, \( b_{10} \) may change significantly. Further accessible parameters from the magnetic geometry are the plateau factor and the effective helical ripple [22]; a more detailed discussion on these parameters is given in Ref. [2]. The plateau factor is closely related to elongation but also includes helical terms in the mod B structure for plateau-type scaling laws such as Lackner-Gottardi scaling [5]. The effective helical ripple is a neoclassical measure for the long-mean-free-path transport of trapped particles. All three configuration describing parameters account for the departure of drift from flux surfaces, but weight the mod B structure differently.

Plotting accessible elongation, effective helical ripples and plateau factors vs. the configuration factor shows correlations (see Fig. 6). A more detailed view shows that, e.g., the radial electric field (e.g. high configuration factor values of W7-A in the upper plot) is to be considered and cannot be described by the elongation. Hence, an inclusion of more relevant transport effects is required. Additional information on the profiles of temperature and densities should be incorporated in order to assess the capabilities of the configuration factor candidates discussed here.

6. Conclusion

Continuing analysis of the International Stellarator Confinement Database has allowed us to refine the conclusions of initial studies by examining the systematics of coherent datasets from the devices W7-AS and LHD.
Bayesian analysis of the W7-AS database confirms the validity of the Connor-Taylor electrostatic Fokker-Planck model for low-beta discharges. The situation is unclear for high beta data. The search for a physical mechanism behind the configuration factors determined in the multi-device ISS04 analysis will require more detailed consideration of profile and neo-classical transport effects, as it is difficult to decide between several possibilities (e.g. helical ripple, elongation etc.) on the basis of the global data included in the database so far.

The ISCDB is an ongoing stellarator community effort. An essential goal aiming at comparative studies is the documentation of global confinement data. Beyond scaling studies, confinement data of further interest are reviewed as well, such as data from operation limits or high beta and parameter scans. For the ISS04 subset of data, the gross parameter scaling of previous studies is confirmed. A clear result is the impact of the magnetic configuration on confinement.

Although this paper did not attempt to extrapolate the existing data to predict future performance, well documented data of subgroups from LHD and Wendelstein 7-AS exist which are compatible with the ITER predictions and typical ELMy H-mode tokamak confinement data in figures of stellarator scaling laws. For a comparison of envisioned tokamak and stellarator reactors, however, further discussion requires the consideration of differences in the operational parameters for burning plasmas, e.g., the density [23].

Beyond documentation and a continuous refinement of data and their quality, the techniques implemented allow for the next steps of ISCDB, i.e. tests of physical models on data and the refinement of possible candidates for a physical configuration factor.
substituting the empirical configuration factor. Extensions of ISCDB with high quality data for scans in the regression variables and configurations are the key for further comparative assessments of the stellarator/heliotron concept.

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References


[21] ITER final design report (summary, document number G A0 FDR 4 01-07-21 R0.4), available at www.iter.org


Footnote

(a) The ISCDB resources are jointly hosted by the National Institute for Fusion Science (NIFS) in Japan and the Max-Planck-Institut für Plamaphysik (IPP) in Germany, see http://iscdb.nifs.ac.jp and http://www.ipp.mpg.de/ISS
log_{10}[\tau_{E} (s)]

log_{10}[\tau^{ISS95} (s)]

ATF
CHS
Heliotron–E
Heliotron–J
HSX
LHD
TJ–II
Wendelstein 7–A
Wendelstein 7–AS

H−mode (DB3v10)
L−mode (DB2v10)
ITER prediction (PPCF41,429(1999))

MS318905 - Fig. 1
$\log_{10}[f \times \tau]_{\text{ISS04}} (\text{s})$

$\log_{10}[\tau]_E (\text{s})$

Symbols:
- △ ATF
- × CHS
- ◀ Heliotron−E
- ◀ Heliotron−J
- ○ HSX
- ○ LHD
- + TJ−II
- ● Wendelstein 7−A
- ○ Wendelstein 7−AS

MS318905 - Fig. 2 (left)
LHD (inward shifted, R = 3.6 m)
LHD (standard, R = 3.75 m)
LHD (outward shifted, R = 3.9 m)
LHD (inward, prolate)
LHD (inward, oblate)
TJ−II
W7−A
W7−AS (high beta)
W7−AS (high iota)
W7−AS (low iota)

\[ f = \left( \frac{\tau_{\text{exp}}}{\tau_{\text{ISS95}}} \right)/1.63 \]
W7–AS HDH Density Scans

- $\tau_{\text{exp}}$ normal confinement
- $\tau_{\text{exp}}$ HDH (attached)
- $\tau_{\text{exp}}$ HDH (detached)

- $f \times \tau_{\text{ISS04}} (f = 1.61)$
- $1.9 \times f \times \tau_{\text{ISS04}} (f = 1.61)$

MS318905 - Fig. 3 (left)
TJ–II – W7–AS Iota Variations (scaling agreement)

\[ \tau \propto \iota^{0.41} a^{2.28} \]

MS318905 - Fig. 3 (right)
MS318905 - Fig. 4
MS318905 - Fig. 5
MS318905 - Fig. 6 (left)
MS318905 - Fig. 6 (center)
MS318905 - Fig. 6 (right)
Fig. 1: Experimental confinement data of stellarators vs. ISS95 scaling. The tokamak data [3] were rephrased employing the plasma elongation and assuming a typical q profile according to Ref. [1]. For the 1998 design of ITER [20], a prediction with uncertainty analysis [4] is plotted against the ISS95 scaling energy confinement value.

Fig. 2: Experimental confinement data of stellarators vs. ISS04 scaling. The right panel is a box-plot of the configuration factors as used for ISS04 [2].

Fig. 3: Examples for parameter scans/variations documented in ISCDB. Left: Energy confinement time in the high-density H mode in Wendelstein 7-AS. Right: Comparison of iota scaling in W7-AS and TJ-II in figures of renormalized confinement times. The f values refer to the ISCDB data subset displayed.

Fig. 4: Configuration factor for different confinement modes and configuration scans in stellarator/heliotron devices.

Fig. 5: Results of Bayesian model comparison of two W7-AS subsets for low ι data. The normalization of collisionality (ν*) is chosen to be the transition of plateau to Pfirsch-Schlüter regime (tokamak like).

Fig. 6: Correlations of elongation κ (left, at 2/3 of effective radius), effective helical ripple ε_{eff} (center) and plateau factor Γ_{stell}/Γ_{tokamak} (right) vs. configuration factor [2].