GRAVITATIONAL WAVE ASTRONOMY —
POTENTIAL AND POSSIBLE REALISATION

J. Hough,* B. J. Meers,* G. P. Newton,* N. A. Robertson,*
H. Ward,* B. F. Schutz,** I. F. Corbett*** and R. W. P. Drever †

*Department of Physics and Astronomy, University of Glasgow,
Glasgow G12 8QQ, Scotland
**Department of Applied Mathematics and Astronomy, University College,
Cardiff, Wales
***Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, U.K.
†California Institute of Technology, Pasadena, California, U.S.A. and
Department of Physics and Astronomy, University of Glasgow,
Glasgow G12 8QQ, Scotland

Abstract

Since the pioneering work of Joseph Weber more than a decade ago there has been a
continuing effort towards the development of more sensitive gravitational wave
detectors. There are a number of interesting astrophysical sources of
gravitational waves including coalescing compact binary star systems, stellar
collapses and rotating neutron stars, and to detect all of these is likely to
require a strain sensitivity better than $10^{-22}$ over a bandwidth of a few hundred
Hz at frequencies at or below 1kHz. To achieve such sensitivity requires
considerable experimental ingenuity; however work in a number of laboratories
suggests that such performance should be attainable using laser interferometry
between freely suspended masses separated by a distance of the order of a
kilometre. This paper includes a review of possible sources and outlines methods
of detection currently being developed or planned, with particular emphasis on
long baseline laser interferometers.

1. INTRODUCTION

The direct detection of gravitational radiation is possibly one of the most
challenging problems in experimental physics at present. Success in this field
will be of considerable significance both for astronomy where new information
about violent interactions in the universe may be obtained, and for physics where
fundamental aspects of relativity theories may be checked.

Experimental research has now been under way in several laboratories around the
world for a number of years. The early experiments of Joseph Weber (e.g. Weber
1970) at the University of Maryland, using resonant aluminium bar detectors at
room temperature, stimulated a range of other experimental searches for
gravitational waves, and while none of these produced confirmed positive results,
they led to the development of new experimental techniques of different types and
to new and unexpected developments in measurement theory (Caves et al. 1980).

As will be clear from the section on sources of gravitational waves, negative
results from these experiments are not surprising in retrospect. However the
experimental developments which followed them are leading to the possibility of
building detectors of sensitivity comparable to theoretically predicted levels of
radiation from various astrophysical sources such as coalescing binary systems,
supernovae collapses and black hole interactions. These detectors would operate
over a frequency range from a few tens of Hz to a few kHz, and make the future for
gravitational wave astronomy look very promising.
Indirect evidence for the existence of gravitational radiation at levels predicted by general relativity has been provided by observations on the binary pulsar PSR 1913+16 (Taylor et al. 1979) and this gives added encouragement to the development of experiments in the field.

1.1 What are gravitational waves?

Soon after Einstein produced his general theory of relativity and gravitation he showed that there are certain solutions of the field equations which satisfy the equations for propagation of waves. These gravitational waves are predicted in Einstein's theory to travel at the speed of light. They are essentially propagating disturbances in the curvature of space-time and they interact with matter in a way somewhat similar to that of travelling tidal forces acting at right angles to the direction of propagation. They are produced by the acceleration of mass (Press and Thorne 1979), just as electromagnetic waves are produced by the acceleration of electric charge. Analogies can be made between gravitational radiation and electromagnetic radiation as described by Maxwell's equations but there are some important and fundamental differences between these phenomena.

Most important is the fact that all mass, unlike electric charge, has the same sign. When a mass is accelerated to generate gravitational waves the matter that recoils to conserve momentum tends to reduce the effect. In other words there is no dipole radiation of gravitational waves. However, a change in shape or quadrupole moment of a system of accelerated masses can take place and does generate gravitational waves. Also, of course, the gravitational interaction is very much weaker than the electromagnetic one and gravitational effects are small unless very large masses or huge accelerations are involved.

Thus gravitational radiation, a wavelike disturbance of space travelling with the velocity of light, and carrying energy, should be produced when matter accelerates in a suitable way.

1.2 How can gravitational waves be detected?

Essentially they can be detected by their property of moving neighbouring inertial reference frames and so neighbouring pieces of free matter relative to each other. The apparent forces acting on pieces of matter, or the motion of free test masses, which may be used to observe the wave, are transverse to the direction of propagation of the wave. If the line joining the masses is perpendicular to this direction then the movement, \( 6L \) say, is proportional to the separation of the masses, \( L \). This makes it convenient to describe the amplitude, \( h \), of a gravitational wave at the detector in terms of the value of \( 6L/L \) induced there in a system of free particles (Press and Thorne 1972, Thorn 1980): \( h = 26L/L \).

In most of the gravity wave experiments performed so far researchers have looked for signals with a predominant frequency in the region of 1 kHz. A typical detector consisted of a single or split aluminium bar for which longitudinal vibrations had a resonant frequency of this order (Weber 1970, Tyson 1973, Douglass et al. 1975, Billing et al. 1975, Drever et al. 1973, Allen and Christodoulides 1975). For short pulses whose energy spectrum peaks close to the
resonant frequency of the bar, the two halves of the bar behave rather like two free test masses and the motions induced can be detected by measuring the strain set up in the bar, which is of the same order of magnitude as $\delta L/L$.

However other experiments use different arrangements, e.g. essentially free test masses placed a distance apart, with laser interferometry as the technique for measuring their relative motion (Moss et al. 1971, Weiss 1972, Winkler 1977, Drever et al. 1977, 1983). The choice of detection method depends to some extent on the frequency of radiation being searched for, the radiation from possible sources spanning a considerable frequency range from $10^{-4}$ Hz for large black hole interactions through $10^{-3} - 10^{-2}$ Hz for fast binary stars to $10^3$ Hz for stellar collapses.

Earth based detectors look very promising from frequencies above several tens of Hz, being limited at low frequencies by the effects of seismic and man made noise. They are aimed at the detection of gravitational waves from sources such as stellar collapses, coalescing binary stars and fast pulsars. Astrophysical models suggest with some confidence that a strain sensitivity of $10^{-22}$ to $10^{-23}$ over a bandwidth of a few hundred Hz should be adequate for the detection of fast pulsed sources, and $10^{-27}$ over a narrow bandwidth would be an interesting level for pulsars.

At very low frequency, 1 cycle/year or lower, pulsar timing is providing interesting limits on the gravitational wave background (Carr 1985, Davies et al. 1985). In this case the pulsar is acting as a very stable clock and the effect of gravitational radiation on the phase of its radio signals is searched for. A stable clock on earth forming the basis of a spacecraft tracking system can also be used for higher frequencies and the Doppler tracking of interplanetary spacecraft has already allowed searches in the mHz frequency range to be made at a gravitational wave amplitude level of $\sim 3 \times 10^{-14}$ (Hellings et al. 1981). The limitation to performance in this case is mainly due to plasma scintillation effects on the radio signals. While the importance of these may be reduced in the future by using dual tracking frequencies, better performance may be expected if optical frequency tracking is used. Then, however, to avoid the effect of the earth's atmosphere the whole system must be spaceborne; and in the long term the performance of laser interferometry between test masses in satellites spaced $10^6$ km or further apart looks much more promising with a potential sensitivity of $\sim 10^{-22}$ at frequencies below 100 mHz (Faller et al. 1983).

While space experiments are exciting for the future, the potential of ground based experiments operating over a frequency range from several tens of Hz to a few kHz seems considerable, and some of the development work for these is most important as a basis for future space experiments. This article will be devoted to aspects of ground based detection of gravitational waves with particular emphasis on the use of laser interferometry between freely suspended test masses spaced by distances of km scale.

2. WHY GRAVITATIONAL WAVE ASTRONOMY?
2.1 Information carried by the waves

This information falls roughly into two categories: testing general relativity and inferring the behaviour of the astrophysical source.
2.1.1 Tests of general relativity. General relativity makes two predictions that are relevant here (Misner et al. 1973, Schutz 1985): that waves travel at the speed of light (massless graviton) and that only two polarisations are allowed, transverse quadrupole (spin 2 graviton). Tests of the wave speed involve comparing arrival times of electromagnetic and gravitational waves from the same event. For example, in a supernova explosion observable electromagnetic radiation probably begins within a few days of the gravitational wave emission. If a supernova at 10 Mpc were detected both optically and gravitationally, with a delay between the two of a few days, then this would establish that their speeds were equal to 1 part in 10^9.

Tests of the polarisation properties of the wave require observations with good signal-to-noise ratio, in which either (i) four detectors register a burst event, or (ii) one detector observes a continuous-wave source, like a pulsar. In general relativity there are five unknowns associated with an incoming wave at any frequency: the amplitudes of two independent polarisations, the phase angle between them (elliptical polarisation), and two angles giving the direction of travel. A detector's response is essentially one number, the strain ΔL/L, plus a measurement of time-of-arrival. For bursts (by which we mean events short compared to a day), detected by four instruments, there is the redundancy necessary for a test: three independent time delays and four values of strain overdetermine the five unknowns. Consistency of these data then tests general relativity's polarisation predictions (Schutz 1986). If continuous sources (gravitational wave amplitude h and polarisation constant for > 1 day) are considered, and if their location is known independently (e.g. a pulsar), then the ratio and times of the maximum and minimum values of the measured strain during a day in a single detector define the wave's polarisation. Once this is found, a full day's data can be fit using just one parameter, the overall amplitude. The goodness of this fit is a test of the polarisation prediction.

A very stringent test of strong field gravity is possible if the coalescence of two black holes from a binary orbit is observed; see section 2.2.2.

2.1.2 Astrophysical information. There are three categories of sources: bursts, continuous wave and stochastic (e.g. a cosmological background). In each case gravitational waves are generated by bulk relativistic motions in strong gravity regions. At observable frequencies (10^2 - 10^4 Hz, lower for pulsars) these correspond to coherent emission regions of size ~ one wavelength, i.e. > 10^9 m. By contrast, electromagnetic observations are possible only at frequencies > 10^7 Hz, and most of our information about potential gravitational wave sources comes from > 10^14 Hz. Coherent photon emission comes from regions typically of atomic size or smaller. Inferences about the large-scale structure of the source therefore require extensive modelling. In some cases this is impossible: supernova electromagnetic observations tell us nothing about the symmetry or asymmetry of the gravitational collapse that triggers the supernova, nothing about whether a neutron star or black hole has been left behind. As another example, pulsar observations have not given us direct evidence of the emission mechanism, the magnetic field alignment, or the spindown mechanism. (If we could directly detect the 60 Hz radio waves from the Crab pulsar we would probably get the answers, but waves of this frequency do not propagate through the interstellar medium). Gravitational waves, in contrast, carry large-scale information, and
their interpretation will be much less model dependent. For this reason, the
impact of their observation on astrophysical models can be expected to be much
more important than one might at first anticipate from the relatively small number
of 'bits' of information the first observations are likely to generate.

Wave bursts from the rapid orbital motion of a close pair of binary neutron stars
or black holes just before they coalesce (as their orbit decays through
gravitational radiation reaction) are the most likely events to be observed and
one of the most fruitful astrophysically. As discussed below, observations by a
network of detectors can give absolute distance to the binary, independently of
any information about the masses of the objects. This allows a nearly model-
independent determination of Hubble's constant, $H_0$ (Schutz 1986) possibly in as
little as a year after the instruments reach their full design sensitivity. The
various electromagnetic determinations of $H_0$ require more complex modelling, and
so are more subject to systematic errors.

Observations of the spectrum, polarisation and amplitude of waves from supernovae
will tell us about the asymmetry of their collapse (and by inference the rotation
of their progenitors and the efficiency of angular momentum transport in
collapse), and observation of the 'ring-down' after a collapse burst will identify
the collapsed object as a black hole or neutron star. Current models of type I
supernovae do not predict collapse to compact objects, although there is
considerable room for doubt. Gravitational radiation observations at the time of
such a supernova could support or contradict these models. It is also possible
that there are many electromagnetically-quiet gravitational collapses, which
might be seen gravitationally. Current astronomical observations allow at least a
factor of two difference between the supernova rate and the pulsar birthrate.

Continuous waves from pulsars account for at least part of the energy loss
required for their observed spindown; observation of these waves would tell us the
asymmetry of the star and the fraction of the spindown due to gravitational waves.
From this one should be able to draw conclusions about surface magnetic field
orientation and strength, and the equation of state of the star material. The
polarisation of pulsar waves tells us the direction in space of the spin axis of
the neutron star, which will help the modelling of pulse formation.

Accreting neutron stars have also been discussed as possible continuous sources.
They may be spun up until they reach the gravitational radiation instability
point, where further accretion of angular momentum is balanced by the radiation of
angular momentum in gravitational waves. Observation of these waves would
constrain a number of things, including most importantly the neutron star equation
of state, because the spin of the marginally stable star is a sensitive function
of this. Determination of the polarisation of the waves would define the star's
spin axis, hence presumably the binary orbit's angle of inclination. This could
determine the masses of the stars if the binary mass function is already known.

The stochastic background of gravitational waves in the detectable frequency
region arises in various theories from 'seeds' for galaxy formation, from
supernovae of population III stars, and from very early fluctuations a few Planck
times after the 'big bang'. We have no other direct information from this epoch,
and observations will provide a very tight constraint on theory. This is one of
the most important areas for gravitational wave observations.
2.2 Some possible sources of gravitational waves

In this section we review the main candidate sources for earth based detectors.

2.2.1 Supernovae. These have been the most frequently discussed sources of gravitational wave bursts, but our ignorance of the initial conditions of the gravitational collapse that initiates them makes it hard to predict their strength with any certainty. We discuss them first because they have in the past been thought to be the best candidates for detection. We shall see in the next section, however, that binary coalescences seem to provide a much greater certainty of detection.

Since the energy flux of a gravitational wave is proportional to the square of the time derivative of its amplitude $h$, there is a simple relation between $h$, the dominant frequency $f$ of the radiation, and the energy $\Delta E$ carried away by the waves in a characteristic time $\tau$ (Misner et al. 1973, Schutz 1985):

$$h = 5.10^{-22} \left( \frac{\Delta E/M_\odot c^2}{10^{-3}} \right)^{1/3} \left( \frac{15\text{Mpc}}{r} \right)^{1/3} \left( \frac{1\text{kHz}}{f} \right)^{1/3} \left( \frac{\text{ms}, \text{sec}}{\tau} \right)^{1/3}$$

Estimates of $f$, $\tau$ and $\Delta E$ vary according to the kind of collapse envisioned. There are nuide upper and lower bounds. The largest conceivable energy release for a one solar mass ($1 M_\odot$) collapse is about 0.2 $M_\odot c^2$, which would require converting into gravitational waves a large fraction of the binding energy released when a neutron star is formed by the collapse. Taking $f = 1/\tau = 1$ kHz gives $h \approx 5.10^{-22}$ from the Virgo cluster (whose distance we take as 15 Mpc). A plausible lower limit comes from assuming that the observed high spatial velocities of pulsars come from asymmetries in the collapses that formed them (Katz 1980, Schutz 1984). This gives $h > 10^{-25}$ for the same $f$ and $\tau$. Within this wide range, certain numerical calculations give more definite predictions. Axisymmetric collapse calculations suggest that if a neutron star is formed then there will be a burst with $\tau \approx 0.5$ ms, $f \approx 5$ kHz, $\Delta E/M_\odot c^2 < 10^{-4}$, giving $h < 5.10^{-22}$ from Virgo (Piran and Stark 1986). If an axisymmetric collapse forms a $10 M_\odot$ black hole, then the calculations suggest $f \approx 1$ kHz, $\tau \approx 1$ ms, $\Delta E/M_\odot c^2 \approx 10^{-3}$, giving $h < 5.10^{-22}$ from Virgo. Actually, axisymmetric numerical calculations sometimes give $\Delta E$ up to 6 times larger than this, but only for rotation rates so rapid that one expects non-axisymmetric instabilities to take over.

For non-axisymmetric collapse, no realistic, fully relativistic numerical calculations are available or seem likely for another five years. One expects that rotational instabilities may deform the star into a rotating bar (Endal and Sofia 1977). Test-particle calculations show that particles falling from circular orbits into rotating black holes can radiate 100 times as much energy as particles falling radially into non-rotating holes (Kojima and Nakamura 1984). Now, the radial-infall test-particle calculation scales up to the case of axisymmetric collapse fairly well (Smarr 1979). If we therefore apply the factor of 100 to the axisymmetric energy rates, we get $h \approx 5.10^{-22}$ for non-axisymmetric neutron star formation. However, this emission may occur over a few rotation periods so $\tau$ may be $\approx 15$ ms, giving $h \approx 1.3 \times 10^{-22}$. This agrees well with independent estimates based on short-lived highly non-axisymmetric rotating configurations (Ipser and
The number of such sources is very uncertain. At the distance of the Virgo Cluster, Type II supernovae may occur as often as once a week with Type I supernovae at a similar rate. Since the pulsar birth rate may be two or three times as high as the supernova rate, there may be a class of electromagnetically quiet 'supernovae' which are nevertheless gravitational wave sources, at a rate that may be as high as one every few days.

Isolated Type II supernovae presumably collapse axisymmetrically, and only go non-axisymmetric if their rotation is rapid enough to excite 'bar-mode' instabilities as they collapse (Endal and Sofia 1977). The Sun, were it to collapse to a neutron star conserving its angular momentum, would have more than enough rotation to excite this instability. Whether cores of pre-supernova stars rotate this fast, and whether their collapses conserve angular momentum, are questions that might only be answered by gravitational wave observations. If Type II supernovae occur in relatively close binary systems, then tidal and mass-exchange effects should guarantee that collapses begin with enough rotation to develop non-axisymmetric instabilities. Moreover, there is some evidence that pre-supernova cores may have non-axisymmetric normal modes that could resonate with the tidal force of an orbiting star, so that the collapse would begin non-axisymmetrically (Das 1985).

Type I supernovae are thought to be accreting white dwarfs that explode thermonuclearly (Wheeler 1982) but they may leave collapsed objects behind (Iser, Labay and Canal 1984). If so, the rotation rates acquired by accretion are in the range needed to excite the bar-mode instability when the star has collapsed: every 0.06 M\(_\odot\) or so of mass accreted by a white dwarf from a disc brings with it enough angular momentum to make a neutron star unstable. If a white dwarf accretes enough mass to go over the Chandrasekhar limit without undergoing a thermonuclear detonation it may simply collapse as an electromagnetically quiet supernova, but would nevertheless have sufficient spin to go non-axisymmetric. Thus, if only one tenth of all collapses in Virgo involve substantial non-axisymmetry, the rate of events at the level of h = 3.10\(^{-22}\) might be as many as 10 per year.

2.2.2 Coalescence of compact-object binaries. This has only recently been recognised as the most reliably detectable source of 'burst' events. When a close binary composed of neutron stars and/or black holes spirals together because of gravitational radiation reaction effects on the orbit, the periodic waves emitted by the orbit in the last 3 seconds before the collision are a clear and easily predicted signal of the event (Peters and Matthews 1963, Clark and Eardley 1977, Kojima and Nakamura 1984) a nearly monochromatic wave train whose frequency 'sweeps' up to a maximum of about 1 kHz for neutron stars and 1 M\(_{\odot}\) \(\sim\) kHz for black holes of mass M\(_{\odot}\) = M/10M\(_\odot\). This sweep of the frequency, or 'chirp', can be predicted with essentially 100% confidence. The sensitivity can therefore be improved by reducing the bandwidth, and can be further improved by observing at that frequency which maximizes the signal to noise, i.e. about 200Hz for detectors using laser interferometry.

The expected signal strength, at frequency f, from a system of total mass M, reduced mass \(\mu\), at a distance r, is
\[ h \approx 1.6 \times 10^{-23} \left( \frac{100 \text{Mpc}}{\tau} \right)^{2/3} \left( \frac{M_{\text{k}}}{M_{\odot}} \right)^{2/3} \left( \frac{f}{200 \text{Hz}} \right)^{2/3} \]

This value of \( h \) is already averaged over source and detector orientations. The time \( \tau \) spent near this frequency is

\[ \tau = \frac{t}{(dt/dt)} \approx 1.2 \left( \frac{200 \text{ Hz}}{f} \right)^{2/3} \left( \frac{M_{\odot}}{M} \right)^{2/3} \left( \frac{M_{\odot}}{\mu} \right) \text{ sec.} \]

A rough estimate of effective signal size obtained after matched filtering in the presence of noise can be obtained by summing the signal amplitude over the square root of the number of cycles of the sweeping waveform. This suggests that for stellar masses \( \sim M_{\odot} \) at a distance of \( \sim 100 \) Mpc signals of \( h \sim 6 \times 10^{-22} \) could be obtained at frequencies of a few hundred Hz. Thus detector sensitivities of \( 10^{-22} \) or better over a bandwidth of a few hundred Hz should be sought.

A plausible estimate of the number of such sources is based on the observation that in our galaxy only one pulsar is in a binary system with a decay time less than a Hubble time (PSR1913+16) (Clark, van den Heuvel and Sutantyo 1979). Combining this with an estimate of the pulsar birth rate gives about \( 7 \times 10^{-7} \) decays per Mpc \(^3\) per year, i.e. 3 events per year out to 100 Mpc or 80 events per year out to 300 Mpc. Although neutron star–neutron star binaries may be more common than ones involving black holes, black hole binaries are stronger emitters and could be seen much further away. Since, in a few years, calculations on supercomputers should give us firm predictions of the signals from such systems, observations of them would give us our most stringent test of strong field general relativity.

Observations can distinguish these various sources from one another. Since \( \tau \) is observable, we can deduce \( M^{2/3}_\mu \), which should give a good idea of the type of system we have. (We expect either neutron stars of \( 1.4M_\odot \) or black holes of \( \sim 10M_\odot \)). Moreover the product \( hr \) depends only on \( \tau \); observations can directly determine the distance to the source (Schutz 1986). This is not quite true for a single detector observation, since \( h \) quoted above is an average over orientations. But if four separate detectors see the event, so that (as described earlier) the source’s location and the wave’s polarisation can be determined, this will tell us all the relevant orientation information, allowing the exact distance to be determined. It is possible that many such events will have supernova-like optical counterparts, which will allow the galaxy in which they occur to be identified and Hubble’s constant to be determined. Failing this, it will still be possible to determine \( H_0 \) statistically, by using the fact that galaxies cluster strongly. If the event rate quoted above is right, the type of detector discussed in the next section, operating in a network, could determine \( H_0 \) to 10% in about a year (Schutz 1986).

Besides these ‘conservative’ sources that arise from ordinary binary evolution,
there may also be more exotic sources: compact-object binaries, for example, formed from Population III stars before the Galaxy formed, residing now in haloes. If only $10^{-4}$ of the Virgo Cluster's mass were in such systems with lifetimes of $10^{10}$ years, events of signal-to-noise ratio $\sim 200$ for proposed detectors would occur more than once per year. Such signals are still too small to have been seen by current prototype detectors.

2.2.3 Pulsars. These are among the most interesting continuous wave sources to search for. Assuming the location and period of the pulsar is known, observers can 'bin' data appropriately over many months to reduce noise. Pulsars are sources of gravitational radiation through any asymmetry they may have about their axis of rotation. Such asymmetries must be present, because of the off-axis magnetic field alignment, but their size depends on such unknowns as the properties of the crust and the surface magnetic field pattern. If we define the ellipticity \( \delta \) of the pulsar to be one minus the ratio of the equatorial semi-minor and semi-major axes, then a pulsar at a distance \( r \) produces waves of amplitude (Zimmerman 1980).

\[
h \sim 10^{-22} \delta \left( \frac{f}{100\text{Hz}} \right)^2 \left( \frac{10\text{kpc}}{r} \right).
\]

at a frequency \( f \) equal to twice its rotating frequency. (There will also be some radiation close to its rotation frequency). For the Crab and Vela pulsars this formula gives

\[
h_{\text{Crab}} \sim 10^{-26} \left( \frac{6}{10^{-5}} \right),
\]

\[
h_{\text{Vela}} \sim 5 \times 10^{-27} \left( \frac{6}{10^{-5}} \right)
\]

Current upper limits on \( \delta \) for these pulsars are of order \( 10^{-3} \), arrived at by assuming that all the spindown is accounted for by energy lost to gravitational waves. For ellipticities approaching $10^2$ to be detectable over integration times of $\sim 10^7$ sec., detector sensitivities of $\sim 10^{-22}$ over a few hundred Hz have to be achieved.

2.2.4 Spinning, accreting neutron stars. These can be strong sources of gravitational radiation if accretion has spun them up to a gravitational radiation ('Friedman-Schutz') instability point, where they go non-axisymmetric and radiate any further accreted angular momentum away in gravitational waves. Because the angular-momentum accretion rate is proportional to the mass accretion rate, the gravitational wave luminosity of such a star accreting from a thin disc will be proportional to its X-ray luminosity (Wagoner 1984). The principal uncertainty is the frequency of the radiation, which depends on that of the relevant unstable normal mode, which is in turn a sensitive function of the equation of state. The marginally stable star probably rotates at least as fast as the millisecond pulsar, i.e. 600 Hz. But the marginally stable mode has zero frequency. The rotation rate of the mode when it builds up enough amplitude to radiate away the accreted angular momentum will be somewhere between zero and 600Hz, probably nearer the lower end. The mode has 4-5 lobes around the equator (not just two as in the familiar bar mode), so the frequency of the radiation will be between zero
and 3 kHz, but 300 Hz is probably not too far wrong. With this we have

\[ h \approx 2.4 \times 10^{-28} \left( \frac{300 \text{Hz}}{f} \right) \left( \frac{F_X}{10^{-10} \text{erg/cm}^2 \text{sec}} \right)^{1/2} \]

where \( F_X \) is the X-ray flux. There are many X-ray sources with fluxes above this level.

Note, however, that with any detector, narrowbanding techniques would have to be used to achieve this sensitivity, and that this is practical only if the frequency is known ahead of time: this could come from the X-rays themselves. Existing X-ray data for bright sources have been searched for periodicities of this type, without success. But the X-ray modulation may be very small, since the required asymmetries of the star are small. (There is in fact a proposal in the US to launch an X-ray telescope dedicated to looking for these periodicities).

2.2.5 The stochastic background. This is the random background of waves produced by all sources. Some of it could be associated with cosmological processes, such as the density fluctuations that led to galaxy formation, cosmic strings, and inflation. If Population III stars were formed, their supernovae would contribute (Bond and Carr 1984). Since stochastic sources are by definition randomly distributed in time and space, it is conventional to describe their energy density in the frequency range \( \Delta f = \theta \) by the ratio \( \Omega_{gw} \) of their energy density to that required to close the universe. This corresponds to mean amplitudes of order

\[ \langle h \rangle = \sqrt{\frac{8}{3}} h(f) \approx 6.1 \times 10^{-26} \left( \frac{\Omega_{gw}}{10^{-10}} \right)^{1/2} \left( \frac{100 \text{Hz}}{f} \right) \]

where \( h(f) \) is the root spectral density of the signal amplitude. Present upper limits on \( \Omega_{gw} \) are typically about \( 10^{-4} \), depending on the spectral region.

For comparison, current cosmic string theory requires \( \Omega_{gw} \approx 10^{-7} \) if strings are to be seeds for galaxy formation (Hogan and Rees 1984, Brandenberger, Albrecht and Turok 1987).

3. GROUND BASED DETECTORS FOR GRAVITATIONAL RADIATION

From the discussion on likely sources of gravitational waves it is clear that detectors with a strain sensitivity of \( 10^{-22} \) or better over a bandwidth of a few hundred Hz are required for significant astrophysical information to be obtained. Ground based detectors of two main types are currently under development — low temperature bar detectors and those using laser interferometry between widely separated test masses.

3.1 Bar detectors

Current work on bar detectors is a continuation of the work started by Joseph Weber in which the two halves of a resonant aluminium bar act as the test masses of the detector, and the internal motion of the bar is sensed by an electro mechanical transducer system (Weber 1970, Tyson 1973, Drever et al. 1973, Douglass
et al. 1975, Billing et al. 1975). The use of low temperature techniques to reduce thermal noise in aluminium bars up to 5 tons in mass is being implemented at a number of places including Stanford University (Michelson and Taber 1984), the University of Louisiana (Hamilton et al. 1986), the University of Western Australia (Veitch et al. 1986) and at CERN (Amaldi et al. 1985) where the University of Rome are mounting a significant effort. In China (Guangzhou) room temperature bars are being developed (Hu En Ke 1985). The Stanford and Rome detectors are already achieving a strain sensitivity of about $10^{-28}$ for millisecond pulses and there are plans to improve the Stanford sensitivity by two orders of magnitude over the next five years and to design a detector system to cover a wide frequency band (Michelson and Taber 1984). In the Stanford detector the motion of the 4.8 x $10^7$ kg aluminium bar, cooled to below 4K, is coupled to that of a resonant diaphragm which modulates the inductance of a coil which is part of a SQUID based flux sensing system. However a sensitivity of better than $10^{-21}$ with these detectors requires bypassing the limit set by the Uncertainty Principle for displacement measurements of the ends of the bar; while there are methods of avoiding this limit by making measurements of a mixture of position and momentum, or by using nonlinear measurements (Caves et al. 1980) it seems likely that broadband operation at the sensitivities ultimately required will be difficult to achieve.

In Japan there is considerable effort being put into the development of resonant low temperature detectors to search for low frequency radiation, in particular that from the Crab and Vela pulsars (Owa et al. 1985).

3.2 Laser Interferometers

3.2.1 Introduction. An alternative approach to detection is to increase the signal size by using freely suspended test masses placed a long distance apart (Forward 1978, Moss et al. 1971, Weiss 1972). Such an arrangement is inherently wideband. To avoid absolute length measurements the test masses can be suspended to give two perpendicular baselines (see Figure 3.2.1) in which the gravitational wave signal will induce a differential displacement. Such a system of orthogonal reference arms is 'tuned' to the quadrupole nature of the gravitational radiation: as the length of one arm increases that of the other decreases and vice versa. The relative length of the two arms can be monitored by laser interferometry.

In order to obtain maximum signal response from a long baseline detector the distance between the masses should be one quarter of the wavelength of the gravitational wave; i.e. for signals of kHz frequency the arm length ($L$) should be close to $10^2$ metres. Such a physical arm length is difficult to envisage for a detector on earth, but effective lengths of the right order can be obtained by folding the light paths in the interferometers, using either optical delay lines or resonant cavities in the arms. A number of prototype detectors of each type are under development. Two of these use optical delay lines (Max Planck Institute, Garching (Shoemaker et al. 1985) where the arm length is 30m, and MIT (Livio et al. 1985) with an arm length of 1.5m) and two use Fabry-Perot cavities (University of Glasgow (Newton et al. 1985, Ward et al. 1985) and California Institute of Technology (Spero 1985) with arm lengths of 10m and 40m respectively). Argon lasers are used for illumination in each case. Currently, the most sensitive of these instruments in terms of strain, (the one at the Max Planck Institute, Garching) has a noise level equivalent to a gravitational wave
amplitude of $10^{-19}$ in a 1 Hz bandwidth at $\sim 2$ kilohertz; the most sensitive instrument in terms of change in relative length of the arms is that at Glasgow which has a noise level equivalent to a change of $1.5 \times 10^{-16}$ m in a 1 Hz bandwidth at $\sim 2$ kHz.

![Diagram of Laser Interferometer]

Figure 3.2.1: Basic Principle of Laser Interferometer

3.2.2 Some limitations and their effect on choice of detector baseline. Proposed gravitational wave experiments are intended to achieve a sensitivity of better than $10^{-22}$ for pulses of a few milliseconds duration and it is clearly essential that the arm lengths of the detector are such that the effect of those background noise displacements which do not scale with length is acceptably small.

The most obvious limitations which first have to be considered are the effects of:
- the Heisenberg Uncertainty Principle
- thermal noise associated with the pendulum modes and internal resonances of the test masses and
- seismic and mechanical noise.

3.2.2.1 Uncertainty Principle. The Uncertainty Principle applied to the test masses ($m$) sets a limit to the amplitude detectable, at unity signal to noise ratio, for pulses of length $\tau$:

$$h = \frac{1}{\sqrt{\hbar}} \sqrt{\frac{m}{k \times \text{km}}} \approx \left( \frac{1 \text{km}}{m} \right) \left( \frac{1000 \text{kg}}{m} \right) \left( \frac{\tau}{10^{-2} \text{sec}} \right)^{1/4} \times 10^{-23}$$

(A similar formula is derived by Edelstein et al. 1978).

Assuming that a reasonable size of test mass is in the range of 10 to 1000 kg for pulses in the range of 1 ms to 10 ms, the minimum arm length must approach 1 km.

3.2.2.2 Thermal Noise Effects. For a test mass suspended as a simple pendulum
the thermal motion associated with the normal mode of the pendulum is peaked at its resonant frequency (approximately 1 Hz) and for frequencies well above this the r.m.s. spectral density of the thermal motion of the mass, m, at frequency f is given by

\[
\left( \frac{4kT f_0}{8\pi^3 f^4 mQ} \right)^{\frac{1}{2}} \text{m/Hz}^{\frac{1}{2}}
\]

where \( kT \) is the thermal energy associated with the resonance of frequency f and quality factor Q (see Weiss 1972). For pulses of length \( \tau \), for which a detection bandwidth of \( \Delta f = 1/2\tau \) centred about a frequency of \( 1/\tau \) is required, the thermal noise of the test masses imposes a limit to gravitational wave amplitude of

\[
h = \left( \frac{1}{\tau} \right) \left( \frac{4kT f_0 \tau^3}{3\pi^3 mQ} \right)^{\frac{1}{2}}
\]

Pendulum quality factors in excess of \( 10^6 \) have been demonstrated in laboratory experiments (Martin 1979) and values > \( 10^8 \) seem possible and test masses of 10 kg and 1000 kg are proposed for searches for 1 msec and 10 msec pulses respectively.

These numbers result in a performance of

\[
h \sim 1.3 \times 10^{-23} \left( \frac{10^8}{Q} \right)^{\frac{1}{2}} \text{ for 1 msec pulses and}
\]

\[
h \sim 4.1 \times 10^{-23} \left( \frac{10^8}{Q} \right)^{\frac{1}{2}} \text{ for 10 msec pulses}
\]

which again suggests that a baseline of 1 km or greater should be chosen.

For the case of the internal resonant modes of the test masses similar considerations show that for a material of density \( \rho \), velocity of sound \( v \) and quality factor Q, the limitation to performance is

\[
h \sim \frac{1}{\mathcal{L}} \left( \frac{5kT}{Qs^3v^3\rho t} \right)^{\frac{1}{2}}
\]

For aluminium, for example, where \( Q \) values of \( \sim 10^6 \) have been demonstrated \( h \) is \( 1.8 \times 10^{-23}/\mathcal{L}(\text{km}) \) for pulses of a few milliseconds duration. A baseline of \( \sim 1 \) km is adequate to allow the proposed performance to be achieved.

From the above considerations it can be seen that the scale of the detector is essentially set, the required arm length being 1 km or greater. It now remains to examine how seismic and mechanical noise will limit the sensitivity of a detector with such an arm length.
3.2.2.3 Seismic Noise. The amplitude and frequency spectrum of seismic and man-made noise is very dependent on the location of the detector site, and in some cases on the time of day. Measurements at likely detector sites suggest that the r.m.s. spectral density of ground movements above a few Hz is approximately

\[
\left( \frac{10^{-7}}{f^2} \right) \text{m/Hz}^\frac{1}{2}
\]

and may be an order of magnitude less if suitable foundations are prepared for the suspensions of the test masses.  
Allowing for the isolation provided by a simple pendulum suspension, which is proportional to \(1/(\text{frequency})^2\),

\[
h \approx 2 \times 10^{-10} \cdot \frac{f_o^2}{\ell (\text{km})} \cdot \left( \frac{1}{10^{-7}} \right)^{1/2}
\]

where \(f_o\) is the pendulum frequency.

\[
h \approx 6 \times 10^{-21} \left( \frac{\ell (\text{km})}{\ell (\text{km})} \right) \left( \frac{1}{10^{-7}} \right)^{1/2} \text{ for } f_o = 1 \text{ Hz}.
\]

It can be seen that a baseline of 1 km will be adequate for 1 msec pulses if a degree of extra isolation of the suspension points of the pendulums of \(\sim 100\) is provided (by passive filters for example). For 10 msec pulses more isolation (a factor of \(10^6\)) is required but should present no serious problem.

3.2.2.4 Choice of Initial Baseline. From the above it is clear that an arm length of 1 km should be adequate to allow the effects of the background noise levels considered to be reduced to an acceptable level. However a longer arm length (of 3–5 km for example) by the same arguments would allow the realisation of enhanced sensitivity, particularly at lower frequencies.

3.2.2.5 Photon Noise Limit. Provided background noise levels can be kept down by suitable choice of baseline etc. it seems likely that the most important limitations to performance will be the statistics of the photoelectron current from the detected light (photon noise) and for a detector where the time spent by the light in the arms is much less than the timescale of the pulses sought, the value set by this is given by:

\[
h = \frac{1}{2} \ell_{\text{eff}} \cdot \frac{\lambda}{2\pi} \cdot \frac{1}{(\text{No of photons in time } \tau)^{1/2}} = \left( \frac{1}{\ell_{\text{eff}}} \right) \left( \frac{\lambda \ell_{\text{opt}}}{2\pi I_0 \tau} \right)^{1/2}
\]

where \(\ell_{\text{eff}}\) is the optical length of each arm, \(\epsilon (\sim 0.5)\) is the efficiency of the photodiode used to detect the light, \(I_0\) is the light power and the other constants have their usual meanings. (A similar formula is derived by Edelstein et al. 1978). However with the availability of very high quality mirrors developed for the laser gyro industry, it is reasonable to expect to choose the number of effective bounces of the light in the arms of the detector to match the timescale
of the pulses. (Note that if the light spends a time longer than half the
gravitational wave period in the arms there may be an actual loss of signal
obtained for some optical arrangements). In this storage time limited case
(effective value of length \(\approx c\tau/2\)) the signal becomes independent of baseline.
The limiting sensitivity for \(\lambda = 514\text{nm}\) (from an Argon laser) and \(\varepsilon \approx 0.5\) is then
set by:

\[
h = \frac{1}{\tau^{3/2}} \left(\frac{\pi \lambda \kappa}{2 I_0 c}\right)^{\frac{1}{2}} \approx 1.7 \times 10^{-21} \left(\frac{10^{-2}}{\tau}\right)^{3/2} \left(\frac{200\text{W}}{I_0}\right)^{1/2}
\]

To achieve an amplitude sensitivity better than \(10^{-22}\) for pulses of several
milliseconds duration requires an input laser power of many kilowatts. However,
with very high quality mirrors, most of the input light will not be lost in the
interferometer but will have been deliberately thrown out to allow the timescales
to be matched. This is clearly wasteful and, as will be explained later, a
technique known as 'optical recycling' (Drever 1983) may be used to recover and
re-use this light, hence reducing considerably the initial laser power required.

3.2.3 Optical Delay Lines. With this arrangement, shown schematically in Figure
3.2.2, the light bounces back and forward between mirrors placed on the test
masses, the geometry of the system being chosen so that the beams do not fall on
top of each other.

![Figure 3.2.2](image)

For a differential change in arm length, such as would be induced by a
gravitational wave, the optical phase difference between the two arms increases
with the number of bounces of the light until the delay time of the light is close
to half a period of the gravitational wave, beyond which point some cancellation
of the signal occurs. Electro-optic modulators can be placed in each arm of the
interferometer as shown. These allow radio frequency modulation techniques to be
used to avoid the effects of excessive intensity noise at low frequency in argon
lasers. The modulators are also used as feedback elements for holding the
interferometer on a null of output intensity at which point the best signal to noise ratio is achieved with an rf modulation scheme. A number of servo systems have to be used to control the orientation of the test masses and to keep the relative lengths of the arms constant at low frequency, but these are not shown here. Initially such a system seems to have much to commend it. For example, because it is a Michelson interferometer it should be independent of frequency fluctuations of the laser if the arm lengths are exactly equal. However initial experiments at the Max Planck Institute (Billing et al. 1983) and independently at the University of Glasgow showed that the effects of light scattered back to the photodiode without completing the full number of traverses considerably increased the sensitivity of such a system to frequency fluctuations. In fact, in the prototype Michelson systems such scattering has proved a considerable problem as a noise source and has been tackled either by frequency stabilising the laser (Billing et al. 1983) or by going in the opposite direction and reducing the coherence of the laser light (Weiss 1982).

In practice, in such a Michelson system, the number of reflections used is limited by one of two factors: the reflection losses at the mirrors, or the total light travel time within each arm of the interferometer. In the case of the very long baseline detectors being proposed, the second factor is more important and it is near optimum to choose the number of reflections to make the light spend a time in each arm equal to the timescale of the gravity wave.

![Diagram](image)

**Figure 3.2.3**

3.2.4 Optical Cavity Interferometers. Another approach to folding the optical paths in the arms of the interferometer, and one which makes the path travelled by the scattered light equal to that travelled by the main beam, is to force all the beams in each arm to lie on top of each other, i.e. to use Fabry Perot cavities in the arms of the interferometer and to monitor the phase changes of the light from these cavities. Such a system was adopted for a prototype detector at the University of Glasgow and more recently at California Institute of Technology, and the principle of the system is shown in Figure 3.2.3 (Drever 1983, Hough et al. 1983).

Light from a single mode laser system passes through a beam-splitter to a pair of Fabry Perot cavities formed between mirrors attached to the test masses. If the lengths of the two cavities are adjusted to give resonance with the light from the
laser, then differential changes in length may be sensed by changes in the
resonance conditions, and small changes in resonance may be detected by measuring
phase changes between the light within each cavity and the input beam, or directly
between one cavity and the other (figure 3.2.3). The relevant phase changes may
be detected by using optical modulation techniques on the light from the cavities
with synchronous demodulation of the detected output light. Such a method
requires a very high level of stabilisation of the frequency of the laser light
with respect to one cavity and of the length of the second cavity to the frequency
of the laser light, and a new scheme for this type of stabilisation was developed
in Glasgow in conjunction with workers at the Joint Institute for Laboratory
Astrophysics in Boulder, Colorado (Drever et al. 1983). This utilises rf phase
modulation of the input light at a frequency outside the bandwidth of the
cavities; the light reflected only from the first mirror of each cavity retains
the modulation while that which has entered the cavity and resonates does not have
the modulation present when it leaks back out in reflection. The total reflected
light then appears amplitude modulated at the frequency of the phase modulation
and the level of amplitude modulation detected synchronously is a measure of the
phase difference between the input light and that in the cavity. This signal can
then be used to control the frequency of the laser by means of an intra-cavity
electro-optic modulator, or by a piezoelectrically driven laser cavity mirror and
an extra cavity modulator (Kerr et al. 1985). In addition it can be used to
control the length of one of the arms by means of a piezoelectrically driven
mirror on one of the test masses or magnetic drive of one of the masses in
conjunction with transducers to move the suspension points of the masses.

3.2.4.1 Sensitivity of Interferometer with Optical Cavities. The maximum
sensitivity for such a system is obtained when the far mirror in each cavity has
maximum reflectivity, R say, and the reflectivity for the input mirror for each
cavity is chosen to give a light storage time equal to the scale of the gravity
wave. In this case when the light reflected from the cavities is interfered to
detect relative phase changes, the photon noise limited sensitivity is the same as
that derived in 3.2.2.5, i.e. for millisecond pulses and an input power of 200W
the resulting amplitude limitation is 1.7 \times 10^{-21}.

The performance is independent of arm length provided the light storage time
condition is met and provided photon noise is the main limitation. However if
mirrors of very low loss are used in the interferometer and optical recycling is
implemented (see Section 3.2.5) the photon noise limited sensitivity can be further
improved to:

\[ h = \left( \frac{4H(1-R)}{2LI_e c r^2} \right)^{1/4} \]

which now varies as \( \sqrt{\text{(arm length)}} \).

3.2.5 The Enhancement of Photon Noise Limited Sensitivity - Optical Recycling
Techniques. Two new methods of enhancing the limit in sensitivity of large
interferometers working at the photon shot noise limit were devised by R.W.P.
Drever (1983). These were stimulated by the fact that as a byproduct of research
on laser gyro's, mirrors of exceedingly low loss (better than 10^{-4}), are becoming
available in the USA and in Europe. One method is relevant for searches for short
gravitational wave pulses and the other for periodic signals such as those from pulsars.

3.2.5.1 Possibility of more efficient use of light in the search for pulses (broadband recycling). In looking for short pulses of gravitational radiation it is close to optimum to arrange the optics in the arms of the interferometer (delay line or Fabry Perot) such that the storage time of the light is equal to the timescale of the gravitational waves. In an optical cavity type system as illustrated in Figure 3.2.3, where high frequency phase modulation techniques are used to allow measurement of the phase difference of the light from the two arms, it can be shown that maximum photon noise limited sensitivity is obtained when optical path differences are adjusted such that the output photodetector observes an interference minimum. In this case if very low loss mirrors are used in the system and the transmissions of the input mirrors of the cavities are chosen to achieve the desired storage time, most of the input light will be directed at the main beam-splitter back towards the laser. An additional mirror introduced in front of the laser can then return most of this light back to the interferometer, with phase adjusted to reinforce the direct light from the laser (see Figure 3.2.4).

![Figure 3.2.4](image)

Auxiliary photodetectors D1, D2 and D4, along with Pockels cells P1, P2 and P3 are indicated as possible ways of monitoring and controlling the phase of the light within the system and the wavelength of the light from the laser. With this arrangement the light intensity within the system as a whole will continue to build up over a time approaching the maximum storage time permitted by losses in the mirrors and other components. However a fast output signal from the beamsplitter is still obtainable. If the main losses are those associated with the cavity mirrors, the end mirrors having maximum reflectivity \( R \), the intensity inside the whole system considered as one large cavity is increased by a factor

\[
\frac{\pi R}{c (1-R)^t}
\]

and the sensitivity is improved to
h = \left( \frac{\lambda c}{2 \pi I_0 c T} \right)^{\frac{1}{2}} \frac{1}{1}

For a baseline of 1 km, currently available mirrors with \(1-R = 10^{-4}\), \(c = 0.5\) and \(I_0 = 200\text{W}\), then \(h = 7 \times 10^{-22} (10^{-3}/T)\).

3.2.5.2 Resonant Enhancement. The standard interferometers described earlier may be used in searches for continuous gravitational wave signals such as those from pulsars or short period binary systems, by applying suitable data analysis techniques. However, for a situation where achievable light storage time is long compared with the period of the signal, and where that period is known, there is a method for obtaining a further improvement in sensitivity. Essentially the storage time for each arm is chosen to be half a period of the gravitational wave signal and the optical system and cavities are so tuned that the light effectively passes back and forth between one arm and the other, with a periodicity equal to that of the gravitational wave (Figure 3.2.5). The optical phase shift due to the mirror motions caused by the gravitational radiation adds up coherently over the storage time of the whole system which may be much longer than the period of the radiation.

![Diagram of resonant enhancement in gravitational wave astronomy]

Figure 3.2.5

The sensitivity of this technique for periodic signals exceeds that of a simple cavity system by a factor approximately equal to the ratio of the overall storage time to the period of the signal. If it is assumed that losses in the system are dominated by mirror losses corresponding to a maximum available reflectivity \(R\), the photon noise limit to sensitivity is given approximately by

\[
h = \left( \frac{\lambda c (1-R)}{2 \pi I_0 T' c} \right)^{\frac{1}{2}} \frac{1}{1}
\]

where \(T'\) is the total duration of the measurement. For a detector of 1 km baseline illuminated with 200 W of light and fitted with mirrors of loss \((1-R) = 10^{-4}\) the photon noise limited amplitude over an integration time of 100
days is

\[ h \approx 2 \times 10^{-28} \]

However, as will be discussed later, other sources of noise may prove to be significant before this performance is achieved.

It should be noted that the sensitivity enhancement techniques are applicable to both multibeam Michelson and Fabry-Perot based systems. In the former case it might appear that very large ultra high quality mirrors are required. However it has been proposed by workers at the Max Planck Institute, Garching, that a large number of individually suspended small mirrors can be used instead. Also Drewer and Weiss (1983) have suggested a frequency tagged system in which the light beams in each arm of the interferometer are allowed to overlap but are frequency shifted on each round trip pass to prevent interference. This is essentially a multibeam Michelson system which can use small mirrors.

3.2.6 Practical Limitations. There are a number of practical limitations which have to be dealt with before the high level of performance required can be reached (Hough et al. 1986). For example, in a system with Fabry-Perot cavities in the arms a vacuum of \(10^{-8}\) torr has to be achieved in the apparatus so that scintillation effects on the light beams are not significant. A very high degree of stability of all the parameters of the laser system (frequency, amplitude and beam geometry) is required but this is expected to be achievable, based on present experience with such systems.

3.3 Possible Experimental Programmes

A number of interferometer systems are planned around the world, two of 4 km arm length in the USA (Drever, Weiss et al. 1985) using either multibeam Michelson or Fabry-Perot techniques, one in Germany of 3 km arm length using a multibeam Michelson system (Winkler et al. 1985), one in Britain with 1 km arms, extendable to 3 km at a later stage, using Fabry-Perot techniques (Hough et al. 1984, 1986), and one in France (Brillet 1985). It is most important to have at least two detectors of this type in the USA and two in Europe to maximise the possibility of detecting gravitational waves and to obtain the maximum possible information from the signals received. However, while many experiments such as locating the direction of pulsed sources require operation with other detectors round the world, it is important that some experiments can be carried out on any instrument in a stand alone way. These could include searches for periodic sources and initial searches for pulses, and the use of cross correlation techniques to look for a continuous background. To this end some groups plan to install a number of separate interferometers in the same vacuum system, some of full length and some of half length. This should allow discrimination against the effects of random outgassing from the walls of the vacuum system and against residual seismic effects and laser fluctuations. One full length and one half length interferometer might be designed to operate in the kHz part of the spectrum and the other full and half length interferometers in the range \(< 200\) Hz. There is room for flexibility in the choice of arrangement here. In Germany the plan is to have eventually 3 separate interferometers around the sides of an equilateral triangle. This would enhance the sensitivity to different polarisations and little sensitivity would be lost by not having the arms at right angles to each
other (Winkler et al. 1985).

3.3.1 Sensitivities. Possible sensitivities obtainable with a detector of 1 km arm length with optical cavities in the arms are shown in figures 3.3.1, 2, 3. The noise levels shown - seismic noise, thermal noise, and the effect of refractive index fluctuations - are based on the following parameters:

Seismic Isolation: passive air mounts, 5 layer antivibration stacks above the suspension points of the masses, and pendulum suspensions.

Thermal noise: suspension Q factor of $10^6$ ($Q_s$)
  test mass Q factor of $10^6$ ($Q_{int}$)

Mirror Loss $1-R = 10^{-4}$
Laser Power = 200W
Pressure in system = $10^{-6}$ torr
Test masses = 1000 kg.

The fundamental limit imposed by the Heisenberg Uncertainty Principle is also shown on the curves.

In Figure 3.3.1 the signals from neutron star binary coalescences - an almost certain source of radiation - at distances of 100 Mpc should be observable at a 10 standard deviation signal to noise ratio. Three events per year might be expected from such events, and the statistical random rate between one half and one full length system would be 1 event per many years if reasonably optimal techniques for detecting the sweeping frequency of the binary signal were used. Three or four such events detected by 4 such instruments might suffice to determine Hubble's constant to ~ 10%. More such events, perhaps several per month, may be expected at a less significant but detectable level. Coincident observations with other detectors would clearly be an important part of these experiments. Coincident observations could also allow the detection of signals from stellar collapses.

Figure 3.3.2 shows that interesting limits could be put on the ellipticity of the Crab pulsar, and a significant search made for any known pulsar of short period and relatively fast slow down rate. Searches for unknown pulsars could be undertaken but the data analysis required is considerable. A search for signals from the spin up of a rotating neutron star, helped by X-ray data from the star, seems a potentially fruitful class of experiment.

From Figure 3.3.3 it is clear that very interesting limits could be put on a continuous background of gravitational waves, a value of $10^{-7}$ of the closure density being detectable at a 3 standard deviation level with cross correlation between two fairly closely positioned detectors (both in the same vacuum envelope at one site or one in Scotland and one in Germany for example).

It is interesting to note from the curves that the internal thermal noise of the test masses is an important factor for both continuous and stochastic experiments and calls for the use of materials of higher internal Q (e.g. sapphire) at some stage, for improved sensitivity.
Figure 3.3.1: Some Burst Sources and relevant noise levels. \( n \) is the number of cycles of the waveform over which the signal can be integrated.
Figure 3.3.2: Signals from sources such as the Crab pulsar, the 1.6 msec pulsar, a hypothetical 2 msec pulsar at the galactic centre, and a spin-up neutron star with an X-ray flux one half that of Sco X-1. An integration time of $10^7$ secs is assumed.
3.3.2 Improvement of Technology. Future developments are possible in almost all areas which apparently limit the performance. For example, the use of photon squeezing techniques may improve the photon noise limited sensitivity achievable with a given light power (Caves 1981, Gia-Banacloche and Leuchs 1987). The use of active suspension and isolation techniques (Robertson et al. 1982, Gia-Banacloche and Stefanini 1986, Rinker and Faller 1984) might improve seismic isolation by a factor of 100 or greater and allow operation at significant levels of sensitivity below 100 Hz provided suspensions and test masses of high enough quality factor are available and a low enough vacuum pressure is attained. Together with the above, the use of mirrors of loss $\approx 2 \times 10^{-5}$, an improvement of suspension $Q$ to $10^9$ and internal $Q$ to $\approx 10^7 - 10^9$, and vacuum pressure to $< 10^{-9}$ torr, should allow an even wider spectrum of experiments to be carried out.
4. CONCLUSION

There is much exciting astrophysical information to be gained from the detection and analysis of gravitational wave signals, and it seems likely that the best way forward involves the setting up of a number of long baseline gravitational wave detectors around the world.

Technological problems are considerable but surmountable and the development effort required should be well rewarded by the results attainable.

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